## LABORATORY VI MOMENTUM

In this lab you will use conservation of momentum to predict the motion of objects motions resulting from collisions. It is often difficult or impossible to obtain enough information for a complete analysis of collisions in terms of forces. Conservation principles can be used to relate the motion of an object before a collision to the motion after a collision, without knowledge of the complicated details of the collision process itself, but conservation of energy alone is usually not enough to predict the outcome. To fully analyze a collision, one must often use both conservation of energy and conservation of momentum.

## Objectives:

Successfully completing this laboratory should enable you to:

- Use conservation of momentum to predict the outcome of interactions between objects.
- Choose a useful system when using conservation of momentum.
- Identify momentum transfer (impulse) when applying energy conservation to real systems.
- Use the principles of conservation of energy and of momentum together as a means of describing the behavior of systems.


## PREPARATION:

Read Serway \& Vuille Chapter 6. You should also be able to:

- Analyze the motion of an object using video analysis tools.
- Calculate the kinetic energy of a moving object.
- Calculate the total energy and total momentum of a system of objects.


## PROBLEM \#1: <br> PERFECTLY INELASTIC COLLISIONS

You have a summer job at NASA with a group designing a docking mechanism that would allow two space shuttles to connect with each other. The mechanism is designed for one shuttle to move carefully into position and dock with a stationary shuttle. Since the shuttles may be carrying different payloads and have consumed different amounts of fuel, their masses may not be identical: the shuttles could be equally massive, the moving shuttle could be more massive, or the stationary shuttle could have a larger mass. Your supervisor wants you to calculate the magnitude and direction of the velocity of the pair of docked shuttles as a function of the initial velocity of the moving shuttle and the mass of each shuttle. You may assume that the total mass of the two shuttles is constant. You decide to model the problem in the lab using carts to check your predictions.

## EQUIPMENT

For this problem you will have several cart weights, a meter stick, a stopwatch, an aluminum track, two PASCO carts, a video camera, and a computer with video analysis applications written in LabVIEW ${ }^{\mathrm{TM}}$ (VideoRECORDER and VideoTOOL). The carts have Velcro pads on one side, which will allow the carts to stick together.

This is the same situation as Problem \#3 in Lab V. If you completed that Problem, you can use that data to check your prediction.

stationary


## Prediction

Write an equation for the final velocity of the stuck-together carts in terms of the cart masses and the initial velocity of cart A. What will be the direction of the final velocity?

Consider the following three cases in which the total mass of the carts is the same $\left(\mathrm{m}_{\mathrm{A}}+\mathrm{mB}=\right.$ constant), where $\mathrm{m}_{\mathrm{A}}$ is the moving cart, and $\mathrm{m}_{\mathrm{B}}$ is the stationary cart:
(a) $\mathrm{m}_{\mathrm{A}}=\mathrm{mB}$
(b) $\mathrm{m}_{\mathrm{A}}>\mathrm{mB}$
(c) $\mathrm{m}_{\mathrm{A}}<\mathrm{mB}_{\mathrm{B}}$

In which case will the final velocity of the carts be the largest? The smallest? Explain your reasoning. Does your answer depend on the initial velocity of cart A?
WARM-UP

Read: Serway \& Vuille Chapter 6, Sections 6.1, 6.2, and 6.3.

1. Draw two pictures, one showing the situation before the collision and the other one after the collision. Is it reasonable to neglect friction? Label the mass of each cart, and draw velocity vectors on each sketch. Define your system. If the carts stick together after the collision, what must be true about their final velocities?
2. Write a momentum conservation equation for this situation and identify all of the terms in the equation. Are there any of these terms that you cannot measure with the equipment at hand?
3. Write down the total energy conservation equation for this situation and identify all the terms in the equation. Are there any of these terms that you cannot measure with the equipment at hand?
4. Which conservation principle should you use to predict the final velocity of the stuck-together carts, or do you need both equations? Explain your reasoning.
5. Solve one of your conservation equations for the magnitude of the final velocity of the carts in terms of the cart masses and the initial velocity of cart $A$. What direction is the final velocity of the carts when $\mathrm{m}_{\mathrm{A}}=\mathrm{mB}_{\mathrm{B}}$ ? When $\mathrm{m}_{\mathrm{A}}>\mathrm{mB}_{\mathrm{B}}$ ? When $\mathrm{m}_{\mathrm{A}}<\mathrm{m}_{\mathrm{B}}$ ?
6. Use the simulation "Lab3Sim" (See Appendix F for a brief explanation of how to use the simulations) to explore the conditions of this problem. For this problem you will want to set the elasticity to zero.

## EXPLORATION

If you have done Problem 3 in Lab. V, you should be able to skip this part analyze the data you already have.

Practice setting the cart into motion so the carts stick together with Velcro after the collision. Try various initial velocities and observe the motion of the carts.

Vary the masses of the carts so that the mass of the initially moving cart covers a range from greater than the mass of the stationary cart to less than the mass the stationary cart while keeping the total mass of the carts the same. Be sure the carts still move freely over the track.

Select the cart masses you will use for $\mathrm{m}_{\mathrm{A}}=\mathrm{mB}_{\mathrm{B}}, \mathrm{m}_{\mathrm{A}}>\mathrm{mB}_{\mathrm{B}}$, and $\mathrm{m}_{\mathrm{A}}<\mathrm{m}_{\mathrm{B}}$ for the same total mass. Determine what initial velocity you will give cart A for each case. Use a stopwatch and meter stick to practice giving cart A these initial velocities.

Set up the camera and tripod to give you the best video of the collision immediately before and after the carts collide. What will you use for a calibration object in your videos? What quantities in your prediction equations do you need to measure with the video analysis software? Is it possible to obtain information before and after the collision with one video analysis, or will you need to analyze each video more than once?

Write down your measurement plan.

## Measurement

If you have done Problem 3 in Lab V, you should be able to skip this part and just use the data you have already taken. Otherwise make the measurements outlined below.

Follow your measurement plan from the Exploration section. Record a video of one collision situation. Use a stopwatch and the distance traveled by the cart before impact with the bumper to estimate the initial velocity of the cart.

Open one your video in VideoTOOL and follow the instructions to acquire data. As a lab group, decide how you will acquire data and analyze the collision. (Will you acquire data for the cart A's motion before the impact and repeat the process for cart A and B after the collision, or will you acquire data for the entire motion of the carts in a single analysis?) Repeat this process for the remaining two collision situations.

Measure and record the masses of the two carts for each situation. Analyze your data as you go along (before making the next video), so you can determine if your initial choice of masses and speeds is sufficient. Collect enough data to convince yourself and others of your conclusions about the efficiency of the collision.


From your videos, determine the velocities of the carts before and after the collision for each situation. Calculate the momentum of the carts before and after the collision. Use your equation from the Warm-up and Prediction questions to calculate the predicted final velocity of the stuck-together carts.

Record the measured and calculated values in an organized data table in your lab journal.

## CONCLUSION

How do your measured and predicted values of the final velocities compare? Compare both magnitude and direction. What are the limitations on the accuracy of your measurements and analysis?

When a moving shuttle collides with a stationary shuttle and they dock (stick together), how does the final velocity depend on the initial velocity of the moving shuttle and the masses of the shuttles? State your results in the most general terms supported by the data.

A collision where kinetic energy is conserved is called "elastic." Any other kind of collision is "inelastic." How can you tell from your data if this collision was elastic, or inelastic?

What conditions must be met for a system's total momentum to be conserved? Describe how these conditions were or were not met for the system you defined in this experiment. What conditions must be met for a system's total energy to be conserved? Describe how those conditions were or were not met for the system you defined in this experiment.

## PROBLEM \#2: ELASTIC COLLISIONS

You are still working for NASA with a group designing a docking mechanism that would allow two space shuttles to connect with each other. The mechanism is designed for one shuttle to move carefully into position and dock with a stationary shuttle. Since the shuttles may be carrying different payloads and have consumed different amounts of fuel, their masses may be different when they dock: the shuttles could be equally massive, the moving shuttle could be more massive, or the stationary shuttle could have a larger mass.

Your supervisor wants you to consider another possible outcome of a shuttle docking. This scenario results in a "Houston we have a problem!" message from the astronauts, and you want to be prepared. In this case, either the pilot misses the docking mechanism or the mechanism fails to function. The shuttles then gently collide and bounce off each other. Your supervisor asks you to calculate the final velocity of each shuttle as a function of the initial velocity of the moving shuttle, the mass of each shuttle, and the fraction of the moving shuttle's initial kinetic energy that is not dissipated during the collision (the "energy efficiency"). You decide to check your calculations in the laboratory using the most efficient bumper you have, a magnetic bumper.

## EQUIPMENT

This is exactly the same situation as for Problem 4 in Lab V. If you did that Problem, you can use that data to check your prediction.

For this problem you will have several cart weights, a meter stick, a stopwatch, an aluminum track, two PASCO carts, a video camera, and a computer with video analysis applications written in LabVIEW ${ }^{\text {TM }}$ (VideoRECORDER and VideoTOOL). The carts have magnets on one side which will allow them to repel each other.


Write an equation for the final velocity of cart B as a function of the initial velocity of cart A and the masses of the two cart s for an elastic collision.

Consider the following three cases in which the total mass of the carts is the same $\left(\mathrm{m}_{\mathrm{A}}+\mathrm{mb}_{\mathrm{B}}=\right.$ constant), where $\mathrm{m}_{\mathrm{A}}$ is the moving cart, and $\mathrm{m}_{\mathrm{B}}$ is the stationary cart:
(a) $\mathrm{mA}_{\mathrm{A}}=\mathrm{mB}$
(b) $\mathrm{m}_{\mathrm{A}}>\mathrm{mB}$
(c) $\mathrm{mA}_{\mathrm{A}}<\mathrm{mB}$

What is the direction of each cart before Based on your measurement from Problem 4 in Lab V or your experience, what is the efficiency of the collision? What does this tell you about your assumption of an elastic collision?
WARM-UP

Read: Serway \& Vuille Chapter 6, Sections 6.1, 6.2, and 6.3.

1. Draw two pictures, one showing the situation before the collision and the other one after the collision. Is it reasonable to neglect friction? Label the mass of each cart, and draw velocity vectors on each sketch. Define your system.
2. Write down the momentum of the system before and after the collision. Is the system's momentum conserved during the collision? Why or why not?
3. If momentum is conserved, write down a momentum conservation equation for the collision. Identify all of the terms in the equation. Is there any momentum transferred into or out of the system? Are you making any approximations?
4. Write down an energy conservation equation for this situation. Identify all the terms in the equation. Is any energy transferred into or out of the system? Are you making any approximations about the efficiency of the magnetic cart bumpers?
5. Use the equations you have written to solve for the final velocity of cart B. Your final velocity of cart B should only depend on the initial velocity of cart A and the masses of the two carts (assuming there is no energy dissipation). Warning: the algebra may quickly become unpleasant! Stay organized.
6. Use the energy conservation equation without any approximations (i.e. include actual efficiency of the bumpers) to calculate the energy dissipation for the collision. Write an expression for the energy dissipated in the collision in terms of the energy efficiency and the initial kinetic energy of the system (Refer back to Laboratory 5 problems 3 and 4.)
7. From your calculations determine the direction of cart A and B after the collision for the three different situations.
8. Use the simulation "Lab3Sim" (See Appendix F for a brief explanation of how to use the simulations) to explore the conditions of this problem. For this problem you will want to set the elasticity to something other than zero.

## EXPLORATION

If you have done Problem 4 in Lab $V$ under conditions of only a small amount of energy dissipation, you should be able to skip this part analyze the data you already have.

Practice setting the cart into motion so the carts bounce apart from the magnetic bumpers. Try various initial velocities and observe the motion of the carts.

Vary the masses of the carts so that the mass of the initially moving cart covers a range from greater than the mass of the stationary cart to less than the mass the stationary cart while keeping the total mass of the carts the same. Be sure the carts still move freely over the track.

To keep the initial kinetic energy approximately the same for different masses of cart A, how should you change the initial velocity of the moving cart? Try it out.

Select the cart masses you will use for $\mathrm{m}_{\mathrm{A}}=\mathrm{m}_{\mathrm{B}}, \mathrm{m}_{\mathrm{A}}>\mathrm{m}_{\mathrm{B}}$, and $\mathrm{m}_{\mathrm{A}}<\mathrm{m}_{\mathrm{B}}$ for the same total mass. Determine an initial velocity for each case that will give you approximately the same initial kinetic energy for cart A. Use a stopwatch and meter stick to practice giving cart A these initial velocities.

Set up the camera and tripod to give you the best video of the collision immediately before and after the carts collide. What will you use for a calibration object in your videos? What quantities in your prediction equations do you need to measure with the video analysis software? Is it possible to obtain information before and after the collision with one video analysis, or will you need to analyze each video more than once?

Write down your measurement plan.

## MEASUREMENT

If you have done Problem 4 in Lab V under conditions of only a small amount of energy dissipation, you should be able to skip this part and just use the data you have already taken. Otherwise make the measurements outlined below.

Follow your measurement plan from the Exploration section. Record a video of one collision situation. Use a stopwatch and the distance traveled by the cart before impact with the bumper to estimate the initial velocity of the cart.

Open one your video in VideoTOOL and follow the instructions to acquire data. As a lab group, decide how you will acquire data and analyze the collision. (Will you acquire data for the cart A's motion before the impact and repeat the process for cart A and B after the collision, or will you acquire data for the entire motion of the carts in a single analysis?) Change the masses of the carts and repeat this process for the remaining two collision situations.

Measure and record the masses of the two carts for each situation. Analyze your data as you go along (before making the next video), so you can determine if your initial choice of masses and speeds is sufficient. Collect enough data to convince yourself and others of your conclusions about the efficiency of the collision.


From your videos, determine the velocities of the carts before and after the collision. Use your equations from the Warm-up and Prediction questions to calculate the final velocity of cart B, the initial and final kinetic energies, and the momentum before and after the collision.

If you have the results from Lab V, Problem \#4, use the appropriate energy efficiency you determined for the collision from that problem. If not, calculate the energy efficiency of each collision from the initial and final kinetic energy of the system. Can you make the approximation that no energy goes into internal energy of the system (energy efficiency $=1$ )?

Record the measured and calculated values in an organized data table in your lab journal.

## CONCLUSION

How do your measured and predicted values of cart B's final velocity compare? What are the limitations on the accuracy of your measurements and analysis? Was the collision perfectly elastic in the three different cases?

Did the directions of the final velocities for cart A and cart B match your prediction? Why or why not? How does the efficiency of the collision affect the conclusion?

What conditions must be met for a system's total momentum to be conserved? Describe how those conditions were or were not met for the system you defined in this experiment. What conditions must be met for a system's total energy to be conserved? Describe how those conditions were or were not met for the system you defined in this experiment.

## PROBLEM \#3: EXPLOSIONS

You have a summer job helping a local ice-dancing group prepare for their season. One routine begins with two skaters standing stationary next to each other. They push away from each other and glide to opposite ends of the ice rink. The choreographer wants them to reach the ends simultaneously. Your assignment is to determine where the couple should stand. To test your ideas you build a model of the situation. Your initial calculation assumes that the frictional force between the ice and the skates can be neglected so you decide to use a metal track and two carts of different masses to test it. Specifically, when the carts push each other apart, you want to find where they should start if they are to reach the ends of the track simultaneously.

## EQUIPMENT

For this problem you will have a meter stick, a stopwatch, an aluminum track, end stops, PASCO carts, and a variety of masses to add to the carts.

A small tip button near one end of a cart releases a plastic arm that can provide the initial push between the two carts.


Calculate a formula for the starting position of the two stationary carts as a function of their masses, and the total distance between the ends of the track if they are to reach the ends of the track simultaneously.

Hint: To make the math easier, you can treat the carts like point masses in your calculation. How does ignoring the cart length affect your result?

## WARM-UP

Read: Serway \& Vuille Chapter 6, Sections 6.1, 6.2, and 6.3. Also review motion in one dimension, sections 2.1 and 2.2

1. Draw a picture that shows the position of the carts when they are stationary. Label the cart masses, the cart lengths, the total length of the track between the end stops, and the distances that each cart must travel to hit the ends of the track from their starting position.
2. Draw another picture that shows the situation just after the carts have pushed off from one another. Label the velocities of the carts.
3. Define your system. Write down the conservation of energy equation for this situation. Identify all of the terms in the equation. (Where does the energy come from that creates the "explosion"?) Are there any terms that you do not know and cannot directly measure?
4. Write down the conservation of momentum equation for this situation. Identify all of the terms in the equation. Are there terms that you do not know and cannot directly measure?
5. Decide which conservation principle will be most useful in this situation. Write down your reason for this decision. Are both useful? Use your conservation equation(s) to determine the relationship between the speeds of the carts.
6. What is an equation that relates horizontal distance, velocity, and time? Write down an expression for the time cart A takes to reach the end of the track in terms of the distance it travels and its velocity. (Hint: to make the calculation easier, you can treat the car as a point mass, ignoring the length of the cart) Write down an expression for the time cart B takes to reach the end of the track in terms of the distance it travels and its velocity. What must be true of the time for cart A and cart B, if they reach the track ends simultaneously?
7. Write down a relationship between the distance traveled by cart A, the distance traveled by cart B, and the total length of the track.
8. Use your equations from questions 5,6 , and 7 to solve for the initial position of the carts before the explosion in terms of their masses and the total length of the track. How does neglecting the length of the carts affect your answer?

## EXPLORATION

Position the carts next to each other on the track and let the side with the tip button of one cart close to the other cart, so that when the button is pushed, the pop up arm can provide the initial push for the two carts. Position the end stops on the aluminum track. How will you tell if the carts hit the end stops at the same time?

Practice pushing the tip button so that your finger will not prevent the cart with the button from moving freely right after you press the button. What is the best way to push the button? Will the contact between your finger and the button affect the motion of the carts? Try it. Make sure the carts move along the track smoothly after you push the button.

Try varying the masses of the carts while keeping the total mass of the carts the same. Be sure the carts still move freely over the track. What masses will you use in your final measurement? Determine a range of cart masses that will give you reliable results.

Write down your measurement plan.

## MEASUREMENT

Position the carts on the track so that when the tip button on one cart is pushed, both carts hit the ends at the same time. Record this position and collect enough data so that you can convince yourself and others that you can predict where the carts should start for any reasonable set of masses.

## ANALYSIS

Where did you need to place the carts so they hit the end at the same time for each case? What is the uncertainty in this measurement?

Use your equation from the Prediction and Warm-up questions to calculate the predicted starting position from the cart masses and the total distance between the end stops for each trial.

## CONCLUSION

How did your measured starting position for the carts compare to your predicted position? What are the limitations on the accuracy of your measurement and analysis?

What will you tell the choreographer? Can you predict where the skaters should be standing for their push off? How does their starting position depend on the mass of the dancers and the length of the ice rink? Does it depend on how hard they push off? State your results in the most general terms supported by your data.

If you have time, modify your prediction equation to include the lengths of the carts. Recalculate the predicted starting position of the carts for each trial. Are the position values closer to your measured values than when you treated the carts like point masses?

## $\boxed{\square}$ CHECK YOUR UNDERSTANDING

1. If a runner speeds up from $2 \mathrm{~m} / \mathrm{s}$ to $8 \mathrm{~m} / \mathrm{s}$, the runner's momentum increases by a factor of
(a) 64 .
(b) 16 .
(c) 8 .
(d) 4 .
(e) 2 .
2. A piece of clay slams into and sticks to an identical piece of clay that is initially at rest. Ignoring friction, what percentage of the initial kinetic energy goes into changing the internal energy of the clay balls?
(a) $0 \%$
(b) $25 \%$
(c) $50 \%$
(d) $75 \%$
(e) There is not enough information to tell.
3. A tennis ball and a lump of clay of equal mass are thrown with equal speeds directly against a brick wall. The lump of clay sticks to the wall and the tennis ball bounces back with one-half its original speed. Which of the following statements is (are) true about the collisions?
(a) During the collision, the clay ball exerts a larger average force on the wall than the tennis ball.
(b) The tennis ball experiences the largest change in momentum.
(c) The clay ball experiences the largest change in momentum.
(d) The tennis ball transfers the most energy to the wall.
(e) The clay ball transfers the most energy to the wall.
4. A golf ball is thrown at a bowling ball so that it hits head on and bounces back. Ignore frictional effects.
a. Just after the collision, which ball has the largest momentum, or are their
 momenta the same? Explain using vector diagrams of the momentum before and after the collisions.
b. Just after the collision, which ball has the largest kinetic energy, or are their kinetic energies the same? Explain your reasoning.
5. A 10 kg sled moves at $10 \mathrm{~m} / \mathrm{s}$. A 20 kg sled moving at $2.5 \mathrm{~m} / \mathrm{s}$ has
(a) $1 / 4$ as much momentum.
(b) $1 / 2$ as much momentum.
(c) twice as much momentum.
(d) four times the momentum.
(e) None of the above.
6. Two cars of equal mass travel in opposite directions with equal speeds on an icy patch of road. They lose control on the essentially frictionless surface, have a head-on collision, and bounce apart.

a. Just after the collision, the velocities of the cars are
(a) zero.
(b) equal to their original velocities.
(c) equal in magnitude and opposite in direction to their original velocities.
(d) less in magnitude and in the same direction as their original velocities.
(e) less in magnitude and opposite in direction to their original velocities.
b. In the type of collision described above, consider the system to consist of both cars. Which of the following can be said about the collision?
(a) The kinetic energy of the system does not change.
(b) The momentum of the system does not change.
(c) Both momentum and kinetic energy of the system do not change.
(d) Neither momentum nor kinetic energy of the system change.
(e) The extent to which momentum and kinetic energy of the system do not change depends on the coefficient of restitution.
7. Ignoring friction and other external forces, which of the following statements is (are) true just after an arrow is shot from a bow?
(a) The forward momentum of the arrow equals that backward momentum of the bow.
(b) The total momentum of the bow and arrow is zero.
(c) The forward speed of the arrow equals the backward speed of the bow.
(d) The total velocity of the bow and arrow is zero.
(e) The kinetic energy of the bow is the same as the kinetic energy of the arrow.

TA Name: $\qquad$

## PHYSICS 1101 LABORATORY REPORT <br> Laboratory VI

Name and ID\#: $\qquad$
Date performed: $\qquad$ Day/Time section meets: $\qquad$
Lab Partners' Names: $\qquad$

Problem \# and Title: $\qquad$
Lab Instructor's Initials: $\qquad$

| Grading Checklist | Points |
| :--- | :--- |
| LABORATORY JOURNAL: |  |
| PREDICTIONS <br> (individual predictions and warm-up completed in journal before each lab session) |  |
| LAB PROCEDURE <br> (measurement plan recorded in journal, tables and graphs made in journal as data is <br> collected, observations written in journal) |  |
| PROBLEM REPORT:* |  |
| ORGANIZATION <br> (clear and readable; logical progression from problem statement through conclusions; <br> pictures provided where necessary; correct grammar and spelling; section headings <br> provided; physics stated correctly) |  |
| DATA AND DATA TABLES <br> (clear and readable; units and assigned uncertainties clearly stated) |  |
| RESULTS <br> (results clearly indicated; correct, logical, and well-organized calculations with uncertainties <br> indicated; scales, labels and uncertainties on graphs; physics stated correctly) |  |
| CONCLUSIONS <br> (comparison to prediction \& theory discussed with physics stated correctly ; possible <br> sources of uncertainties identified; attention called to experimental problems) |  |
| TOTAL(incorrect or missing statement of physics will result in a maximum of $60 \%$ of the <br> total points achieved; incorrect grammar or spelling will result in a maximum of 70\% of <br> the total points achieved) |  |
| BONUS POINTS FOR TEAMWORK <br> (as specified by course policy) |  |

* An "R" in the points column means to rewrite that section only and return it to your lab instructor within two days of the return of the report to you.

