## Appendix C: Graphing

The graph is one of the most powerful tools for data presentation and data analysis. When used properly, graphs become important guides that illustrate how an experiment was conducted. It is important to graph your data as you take it. Once you finish data taking, graphs are also the easiest way to make sense out of data.

LabVIEW ${ }^{\text {TM }}$ will often provide you with a graph, but on occasion you will have to make your own. Good graphing practice, as outlined below, is a way to save time and effort while solving a problem in the laboratory.

## How do I make a graph?

1. Graph paper was invented to make it easy to make graphs. Even if you are just making a quick sketch for yourself, it will save you time and effort to use graph paper. That is why your lab journal pages are graph paper. Make sure you graph your data as you take it.
2. Every graph should have a title so that later you can recognize quickly what data it represents. In a stack of graphs, it is difficult to keep one graph distinct from another without a clear, concise title.
3. The axes of the graph should take up at least half a page. Give yourself plenty of space so that you can see the pattern of the data as it is developing. Both axes should be labeled so that later you will know what quantity was being measured and in what units it was measured without a lot of effort.
4. The scales on the graphs should be chosen so that the data occupies most of the space of your graph. "Fill up the graphs!" You do not need to include zero on your scales unless it is important to the interpretation of the graph.
5. If you have more than one set of data on a set of axes, be sure to label each set so you can't get confused later.

## How do I plot data and its uncertainty?

Another tool that makes data analysis easier is a table to record all of your data. A useful data table starts with a title and column headings so that you will not forget what all the numbers mean. The column headings are usually the labels for the axes of your graph. As an example look at Table C-1 and Graph C-1 on the next page where a position-versus-time graph is drawn for a hypothetical situation.

The uncertainty is shown on the graphs as a line representing a range of possible data with the principal value at the center. The lines are called error bars, and they are useful in determining if your data agrees with your prediction. Any curve (function) which represents your data should pass through your error bars not the data points.

Table C-1:
Position vs. Time: Exercise 3, Run 1

| Time/sec | Position/cm |
| :---: | :---: |
| 0.1 | $10 \pm 5$ |
| 0.2 | $20 \pm 5$ |
| 0.3 | $30 \pm 5$ |


| 0.4 | $39 \pm 5$ |
| :--- | :--- |
| 0.5 | $49 \pm 5$ |
| 0.6 | $59 \pm 5$ |
| 0.7 | $69 \pm 5$ |
| 0.8 | $79 \pm 5$ |
| 0.9 | $89 \pm 5$ |
| 1.0 | $98 \pm 5$ |

Graph C-1:


## What is a best-fit straight line?

Whenever you have a graph of data, you are often asked to determine the relationship between two quantities that you measure (i.e. position and time). Do not connect the dots that are your data points; instead draw a smooth curve near all of the data, but within the error bars. If you expect it to be linear, then you should use a clear straight edge and try to draw the line. The line should pass through all of the error bars and have as many principal values beneath it as above it. The line does not, however, need to touch any of the data points.

When done correctly, this straight line represents the function that best fits your data. You can read from your graph the slope and intercept of the line. These quantities usually have some important physics interpretation. Some computer programs, like Excel or Cricket Graph, will determine the best straight line for your data and compute its slope, intercept, and their uncertainties. You should check with your lab instructor, before you use a graphing program to see if it is appropriate. As an example, look at Graph C-1. The dotted lines are possible linear fits, but the solid line is the best. Once you have found the best-fit line, you should determine the slope from the graph and record its value.

## How do I find the slope of a line?

The slope of a line is defined to be the ratio of the change in a line's ordinate to the change in the abscissa, or the change in "rise" divided by the change in "run." Your text explains slope. For Graph C-1, the slope of the best-fit line will be the change in the position of the object divided by the time interval for that change in position.

To find the value of the slope, look carefully at your best fit line to find points along the line that have coordinates that you can identify. It is usually not a good idea to use your data points for these values, since your line might not pass through them exactly. For example, the best fit line on Graph C-1 passes through the points
( $0.15 \mathrm{sec}, 15 \mathrm{~cm}$ ) and ( $1.10 \mathrm{sec}, 110 \mathrm{~cm}$ ). This means the slope of the line is:

$$
\begin{aligned}
\text { slope } & =\frac{\text { change in "rise" }}{\text { change in "run" }} \\
& =\frac{110 \mathrm{~cm}-15 \mathrm{~cm}}{1.10 \mathrm{sec}-0.15 \mathrm{sec}} \\
& =\frac{95 \mathrm{~cm}}{0.95 \mathrm{sec}} \\
& =100 \frac{\mathrm{~cm}}{\mathrm{sec}}
\end{aligned}
$$

Note that the slope of a position-versus-time graph has the units of velocity.

## How do I find the uncertainty in the slope of a line?

Look at the dotted lines in Graph C-1. These lines are the largest and smallest values of the slopes that can realistically fit the data. The lines run through the extremes in the uncertainties and they represent the largest and smallest possible slopes for lines that fit the data. You can extend these lines and compute their slopes. These are your uncertainties in the determination of the slope. In this case, it would be the uncertainty in the velocity.

## How do I get the slope of a curve that is not a straight line?

The tangent to a point on a smooth curve is just the slope of the curve at that point. If the curve is not a straight line, the slope depends where on the curve you measure it.

To draw a tangent line at any point on a smooth curve, draw a straight line that only touches the curve at the point of interest, without going inside of the curve. What you are trying to do is get an equal amount of space between the curve and the tangent line on both sides of the point of interest.

The tangent line that you draw needs to be long enough to allow you to easily determine its slope. You will also need to determine the uncertainty in the slope of the tangent line by considering all other possible tangent lines and selecting the ones with the largest and smallest slopes. The slopes of these lines will give the uncertainty in the tangent line. Notice that this is exactly like finding the uncertainty of the slope of a straight line, but requires more imagination.

## How do I "linearize" my data?

A straight-line graph is the easiest graph to interpret. By seeing if the slope is positive, negative, or zero you can quickly determine the relationship between two measurements. Unfortunately, not all the relationships in nature are straight lines. However, if we have a theory that predicts how one measured quantity (e.g. position) depends on another (e.g. time) for the experiment, we can make the graph be a straight line. To do this, you make a graph with the appropriate function of one quantity on one axis (e.g. time squared) and the other quantity (e.g. position) on the other axis. This is what is called "linearizing" the data.

For example, if a rolling cart undergoes constant acceleration, the position-versus-time graph is a curve. In fact, our theory tells us that the curve should be a parabola. To be concrete we will assume that your data starts at a time when the initial velocity of the cart was zero. The theory predicts that the motion is described by $x=0.5 a t^{2}$. To linearize this data, you square the time and plot position versus time squared. This graph should be a straight line with a slope of 0.5 a. Notice that you can only linearize data if you know, or can guess, the relationship between the measured quantities involved.

## How do I interpret graphs from LabVIEW ${ }^{\text {TM }}$ ?

Graphs C-2 through C-4 were produced with LabVIEW ${ }^{\text {™ }}$. They give the horizontal position as a function of time. Even though LabVIEW ${ }^{\top}{ }^{\top}$ draws the graphs for you, it usually does not
label the axes and never puts in the uncertainty. You must add these to the graphs yourself.

One way to estimate the uncertainty is to observe how much the data points are scattered from a smooth behavior. By estimating the average scatter, you have a fair estimate of the data's uncertainty. Graph C-2 shows position versus time for a moving cart. The vertical axis is position in cm and the horizontal axis is time in seconds. See if you can label the axes appropriately and estimate the uncertainty for the data.

## Graph C-2



Finding the slope of any given curve with LabVIEW ${ }^{\text {TM }}$ should be easy after you have chosen the mathematical equation of the best-fit curve. Graphs C-3 and C-4 each show one possible line to describe the same data. From looking at these graphs, the estimate of the data's uncertainty is less than the diameter of the circles representing that data.

## Graph C-3



Graph C-4
H-Motion Plot


The line touching the data points in Graph C-3 has a slope of $0.38 \mathrm{~m} / \mathrm{sec}$. This is the minimum possible slope for a line that fits the given data.

The line touching the data points in Graph C-4 shows a slope of $0.44 \mathrm{~m} / \mathrm{sec}$. This is the maximum possible slope for a line that fits the given data.
You can estimate the uncertainty by calculating the average difference of these two slopes from that of the best-fit line. Therefore the uncertainty determined by graphs C-3 and C-4 is $\pm 0.03 \mathrm{~m} / \mathrm{sec}$.

## PRACTICE EXERCISES:

Explain what is wrong with each of the graphs below.



