Waves Chap 16

An example of
Dynamics
Conservation of Energy

Conceptual starting point
Forces
Energy

## WAVES

So far this quarter

Conservation theories

| | | |
|---|---|---|
| mass | m | scalar |
| energy | E | scalar |
| momentum | $\vec{p}$ | vector |
| angular momentum | $\vec{L}$ | vector |

All conservations theories:
Conservation of X

$$X_{system\ f} - X_{system\ i} = X_{input} - X_{output}$$

$$\Delta X_{system} = \Delta X_{transfer}$$

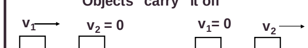
So far we have focused on the properties of the system

i.e. How does the system energy change?

Now we begin a more detailed study of
Energy Transfer Mechanisms
How can energy be transferred into or out of a system?
How can energy be transferred from one system to another?

Mechanisms of energy transfer across a distance

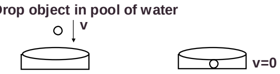
Objects “carry” it off




before elastic collision
after elastic collision

No object “carries” it off

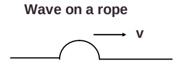
Drop object in pool of water




Where does energy go?

Waves transport energy across a distance
without transporting matter across that distance.

Examples:
Wave on a rope

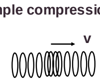


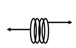

The pulse moves with speed v
but no object, or part of an object, moves with that speed

Parts of the rope move up and down,
the wave moves horizontally

Transverse wave
Easiest to visualize
Most examples

Example compression wave in a spring:



Parts of the spring move back and forth,
the pulse moves horizontally

Longitudinal Wave

You will explore the properties of both types of waves in the laboratory.

## Wave Equation

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{T_o}{\rho_o}\frac{\partial^2 \psi}{\partial z^2}$$

Notation in the textbook $\frac{\partial^2 y}{\partial t^2} = \frac{F}{\mu}\frac{\partial^2 y}{\partial x^2}$

Solution

What function?

Second derivative with respect to time gives same function as second derivative with respect to position.

Since position and time are different quantities, choose each function so that the second derivative is the same as the function.

$$\frac{\partial^2 y}{\partial t^2}=\frac{F}{\mu}\frac{\partial^2 y}{\partial x^2}$$

try $y = A\cos(ax+b)\cos(ct+d)$

$$\frac{\partial y}{\partial t} = -A\cos(ax+b)c\sin(ct+d)$$

$$\frac{\partial^2 y}{\partial t^2} = -A\cos(ax+b)c^2\cos(ct+d)$$

$$\frac{\partial y}{\partial x} = -Aa\sin(ax+b)\cos(ct+d)$$

$$\frac{\partial^2 y}{\partial x^2} = -Aa^2\cos(ax+b)\cos(ct+d)$$

$$-A\cos(ax+b)c^2\cos(ct+d) = -\frac{F}{\mu}Aa^2\cos(ax+b)\cos(ct+d)$$

$$c^2 = \frac{F}{\mu}a^2$$

$$\frac{c^2}{a^2} = \frac{F}{\mu}$$

Need another way to find either a or c.

---

$y = A\cos(ax+b)\cos(ct+d)$

At what position does this repeat?
Wavelength

$y_1 = A\cos(ax_1+b)\cos(ct+d)$

$y_2 = A\cos(ax_2+b)\cos(ct+d)$

$y_1 = y_2$

$$A\cos(ax_1+b)\cos(ct+d) = A\cos(ax_2+b)\cos(ct+d)$$

$$\cos(ax_1+b) = \cos(ax_2+b)$$

$$ax_1+b+2\pi = ax_2+b$$

$$2\pi = ax_2 - ax_1$$

$$2\pi = a(x_2-x_1) = a\lambda$$

$$\frac{2\pi}{\lambda} = a$$

At what time does this repeat?

---

At what time does this repeat?
Period

$y_1 = A\cos(ax+b)\cos(ct_1+d)$

$y_2 = A\cos(ax+b)\cos(ct_2+d)$

$y_1 = y_2$

$$A\cos(ax+b)\cos(ct_1+d) = A\cos(ax+b)\cos(ct_2+d)$$

$$\cos(ct_1+d) = \cos(ct_2+d)$$

$$ct_1+d+2\pi = ct_2+d$$

$$2\pi = ct_2 - ct_1$$

$$2\pi = c(t_2-t_1) = c\tau$$

$$\frac{2\pi}{\tau} = c$$

---

$$\frac{\partial^2 y}{\partial t^2}=\frac{F}{\mu}\frac{\partial^2 y}{\partial x^2}$$

$$y = A\cos(ax+b)\cos(ct+d)$$

$$\frac{c^2}{a^2} = \frac{F}{\mu}$$

$$\frac{2\pi}{\lambda} = a$$

$$\frac{2\pi}{\tau} = c$$

$$\frac{\left(\frac{2\pi}{\tau}\right)^2}{\left(\frac{2\pi}{\lambda}\right)^2} = \frac{F}{\mu}$$

$$\frac{(\lambda)^2}{(\tau)^2} = \frac{F}{\mu} \qquad \frac{\lambda}{\tau} = v$$

$$v^2 = \frac{F}{\mu}$$

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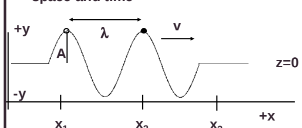
Waves

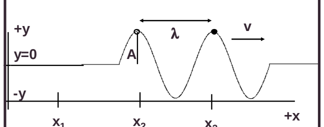
Energy transport through a medium

TRAVELING WAVES

Our normal observations are in both space and time




$x_2 - x_1 = \lambda$

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The wave pattern repeats over a distance λ

λ is called the wavelength

If the wave takes a time τ

to travel a distance λ

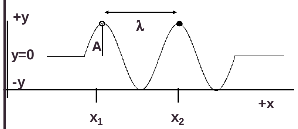
τ is called the period

Its speed is $v = \lambda / \tau$

$$\lambda = v\tau$$

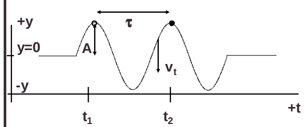
Consider two different simple observations

Observe at fixed time (snapshot)




$x_2 - x_1 = \lambda$

Observe at a fixed position




$t_2 - t_1 = \tau$

Wave velocity v

Determined by properties of medium

Restoring force

Mass density

Wavelength λ

Distance of pulse repetition

Period τ

Time for pulse repetition

For a wave

$\lambda = v\tau$

or

$f = 1/\tau$

$\lambda f = v$

Material is moving in oscillations

"Spring" forces in material

$F = -k\Delta x$

Velocity of material $v_t$

Perpendicular to wave motion

transverse wave

Parallel (and anti-parallel) to wave motion

longitudinal wave

Oscillating motion

Wave amplitude A

Maximum displacement of medium

Angular frequency $2\pi f = \omega$

For a spring (oscillating system)

How does the wave speed depend on the properties of the system (spring)?

Some "intuitive" reasoning which could be wrong (check it with nature in the lab)

Frequency depends on

mass and spring constant

$$\text{frequency} = \sqrt{\frac{k}{m}}$$

$$\text{frequency} = \sqrt{\frac{\text{restoring force strength}}{\text{mass term}}}$$

For wave motion

Frequency is determined by wave velocity

$\lambda f = v$

It is reasonable that

wave velocity goes like

$$\sqrt{\frac{\text{restoring force strength}}{\text{mass term}}}$$

$$\sqrt{\frac{\text{restoring force strength}}{\text{mass term}}}$$

want units to be velocity

$$\sqrt{\frac{k}{m}} \quad \text{units} \quad \sqrt{\frac{\text{force / length}}{\text{mass}}} = \frac{1}{\text{time}}$$

to get units of velocity

multiply by length

$$\frac{\text{length}}{\text{time}} = \sqrt{\frac{\text{force} \bullet \text{length}^2 / \text{length}}{\text{mass}}}$$

$$\frac{\text{length}}{\text{time}} = \sqrt{\frac{\text{force}}{\text{mass / length}}}$$

Guess

$v = \sqrt{\frac{T}{\mu}}$ $\mu$ = mass / length

Wave phenomena

Determined by boundary conditions

An important example:

standing waves

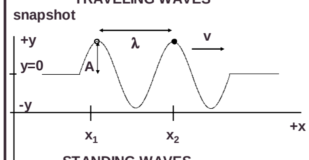
Waves that seem to be

fixed in space

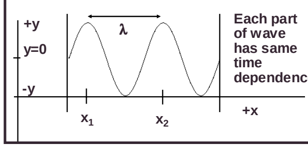
## STANDING WAVES

Waves as oscillations

### TRAVELING WAVES




### STANDING WAVES




Each part of wave has same time dependence

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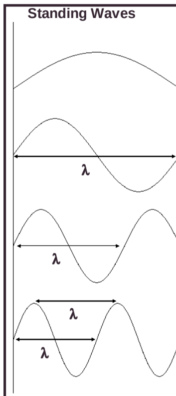
Standing Waves




fixed supports

To keep the same "shape" over the whole rope

Each piece of the rope must oscillate up and down at the same rate.

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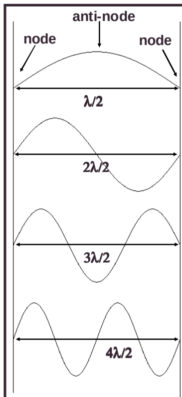


nodes

Standing wave has zero amplitude

anti-nodes

Standing wave has maximum amplitude

$L = n\lambda/2$

for a standing wave

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A standing wave is described by

$y = f(x)\, g(t)$

Time dependence of every position of rope is the same.

Each point on rope is moving up and down. Harmonic motion

Periodic in time

Shape of wave is periodic.

Periodic in space

$y = 0$ at $x = 0$, $y = 0$ at $x = L$, $y = 0$ at nodes

Try

$y = A \sin(kx) \cos(\omega t)$

$k = 2\pi/\lambda \qquad \omega = 2\pi/\tau$

Where does = 0

When $x = n\lambda/2$

a node

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Example:

While watching a concert you wonder about the physics of playing a violin. One of the violin strings sounds an A note (440 Hz) when played without fingering. The string is 30 cm long and has a mass of 2.0 g. Where must one put one's finger to play a C note (528 Hz) with the same string?

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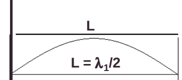
A finger placed on a string becomes a node.

Geometry of standing wave gives wavelength.

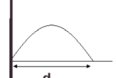
Frequency related to wavelength by speed

Same string, same wave speed.




No fingering

$f_1 = 440$ Hz




Finger at a node

$f_2 = 528$ Hz

$d = \lambda_2/2$

Find distance of finger from end of string d

Use:

$\lambda = v\tau \qquad \tau = 1/f$

$d = \lambda_2/2$ need $\lambda_2$

$\lambda_2 = v\tau_2$

$\lambda_2 = v/f_2$

$d = v/2f_2$ need v

From string without fingering

$\lambda_1 = v/f_1$

$\lambda_1 f_1 = v$

$d = \lambda_1 f_1/2f_2$ need $\lambda_1$

$L = \lambda_1/2$

$\boxed{d = Lf_1/f_2}$

d = (30 cm) (440 Hz)/(528 Hz)

$\boxed{d = 25\text{ cm}}$

Units are correct for a length

Finger position is reasonable,
it is less than the length of the string.

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Example:

You are helping a friend design a high tech electric violin. She wants one steel string to vibrate at a frequency of 512 Hz in its lowest mode. The next string should be made of the same material and have a lowest mode of 256 Hz. If the tensions of the two strings are different, they will warp the instrument. To decide on the thickness of the string, she asks You to calculate the ratio of the mass per unit length of the two strings so that they have the same tension.

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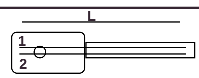
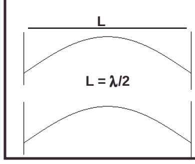



$f_1 = 512$ Hz
$f_2 = 256$ Hz

string tension $T_1 = T_2 = T$
mass per unit length

What is $\mu_2/\mu_1$ ? $\mu_1$ , $\mu_2$

Vibration of strings is standing wave.
Lowest mode - nodes only at ends.
Mass per unit length and tension give wave speed
Wavelength and frequency gives speed
Geometry gives wavelength for standing wave




string 1

$\lambda_1 = \lambda_2 = \lambda$

string 2

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Use:

Wave properties

$\lambda = v\tau$ $\tau = 1/f$

$$v = \sqrt{\frac{\text{Tension}}{\text{mass density}}}$$

$$v = \sqrt{\frac{T}{\mu}}$$

For the two strings with the same tension

$$\mu_1 = \frac{T}{v_1^2}$$

$$\mu_2 = \frac{T}{v_2^2}$$

$$\frac{\mu_2}{\mu_1} = \frac{v_1^2}{v_2^2}$$ need $v_1$, $v_2$

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$$\frac{\mu_2}{\mu_1} = \frac{v_1^2}{v_2^2}$$

$\lambda = v_1\tau_1$

$\lambda/\tau_1 = v_1$

$\lambda = v_2\tau_2$

$\lambda/\tau_2 = v_2$

$$\frac{\mu_2}{\mu_1} = \frac{\left(\frac{\lambda}{\tau_1}\right)^2}{\left(\frac{\lambda}{\tau_2}\right)^2} = \frac{\left(\frac{1}{\tau_1}\right)^2}{\left(\frac{1}{\tau_2}\right)^2}$$

$$\boxed{\frac{\mu_2}{\mu_1} = \frac{f_1^2}{f_2^2}}$$

$$\frac{\mu_2}{\mu_1} = \frac{(512\text{Hz})^2}{(256\text{Hz})^2} = \frac{(2\times 256\text{Hz})^2}{(256\text{Hz})^2} = 4$$

Units are correct since a ratio has no units

The lower frequency string has a larger mass so it vibrates slower at the same tension.

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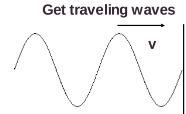
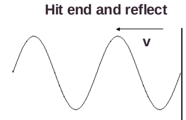
What is the mechanism for getting Standing Waves ?

Shake one end of a rope and keep shaking it.

Get traveling waves

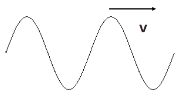



Hit end and reflect

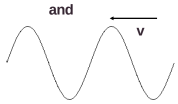


But waves are still coming in

Combine




and




At each value of x, add the y from the incoming wave and the y from the outgoing wave.

Superposition

Result depends on how the waves match

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How the waves match depends on

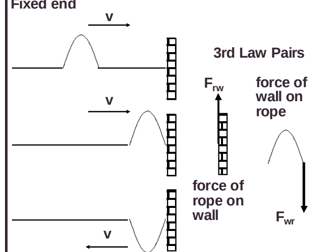
Their Reflection Properties

Two cases

Fixed end

Free end

Fixed end




Reflections from a fixed end are inverted

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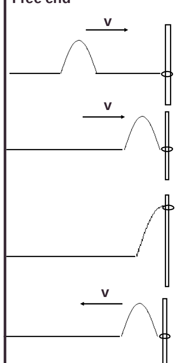
Free end




Reflections from a free end are not inverted

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Question

At t=0, your friend claps hands, with 1.00 second between each clap. You are standing 550 ft away from your friend. The speed of sound in air is 1100 ft/s.

When do you hear the first clap?

What is the time interval you hear between the first clap and the second clap?

At t=0, your friend claps hands, with 1.00 second between each clap. You are 550 ft away from your friend and running away with a constant speed of 2 ft/s. The speed of sound in air is 1100 ft/s.

When do you hear the first clap?

What is the time interval you hear between the first clap and the second clap?

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At t=0, your friend claps hands, with 1.00 second between each clap. You are standing 550 ft away from your friend. The speed of sound in air is 1100 ft/s.

When do you hear the first clap?

Time for sound to travel 550 ft

$$v_s = \frac{d}{t}$$

$$t = \frac{550 \quad ft}{1100 \quad ft/s} = 0.500s$$

What is the time interval you hear between the first clap and the second clap?

Time interval 1.00 s

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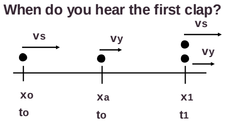
At t=0, your friend claps hands, with 1.00 second between each clap. You are 550 ft away from your friend and running away with a constant speed of 2 ft/s. The speed of sound in air is 1100 ft/s.

When do you hear the first clap?




to = 0

xo = 0

xa = 550 ft

t1 = ?

x1 = ?

vs = 1100 ft/s

vy = 3 ft/s

$$v_s = \frac{x_1}{t_1} \qquad v_y = \frac{x_1 - x_a}{t_1}$$

2 unknowns, 2 equations

$$v_s t_1 = x_1$$

$$v_y = \frac{v_s t_1 - x_a}{t_1}$$

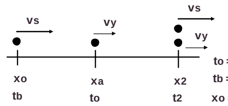
$$x_a = (v_s - v_y) t_1$$

$$\frac{x_a}{(v_s - v_y)} = t_1$$

$$\frac{550 \quad ft}{1097 \quad ft/s} = t_1$$

0.501 s = $t_1$

**What is the time interval you hear between the first clap and the second clap?**




to = 0
tb = 1 s
xo = 0
xa = 550 ft
t2 = ?
x2 = ?
vs = 1100 ft/s
vy = 3 ft/s

$$v_s = \frac{x_2}{t_2 - t_b} \qquad v_y = \frac{x_2 - x_a}{t_2}$$

2 unknows, 2 equations

$$v_s(t_2 - t_b) = x_2$$

$$v_y = \frac{v_s(t_2 - t_b) - x_a}{t_2}$$

$$v_y t_2 + x_a = v_s(t_2 - t_b)$$

$$x_a + v_s t_b = (v_s - v_y)t_2$$

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$$\frac{x_a + v_s t_b}{(v_s - v_y)} = t_2$$

time interval between the first & second clap

$$\Delta t = t_2 - t_1$$

$$\frac{x_a}{(v_s - v_y)} = t_1$$

$$\frac{x_a + v_s t_b}{(v_s - v_y)} - \frac{x_a}{(v_s - v_y)} = \Delta t$$

$$\frac{v_s}{(v_s - v_y)} t_b = \Delta t$$

$$\frac{1100 \ \text{ft/s}}{1097 \ \text{ft/s}} = \Delta t$$

$1.003 \text{ s} = \Delta t$

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**Or using waves**

**Emitted wave (claps)**

$$v_s = \frac{\lambda}{T} \qquad T = t_b$$

**Received wave (heard claps)**

$$v' = \frac{\lambda'}{T'}$$

$T' = \Delta t$

$\lambda' = \lambda$

$v' = v_s - v_y$

$$v_s - v_y = \frac{v_s t_b}{\Delta t}$$

$$\frac{v_s}{v_s - v_y} t_b = \Delta t$$

same as before

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**Example:**

While trying to get home for spring break, you have a flat tire on the freeway. As you change your tire a truck approaches you blowing his air horn which has a frequency of 250 Hz. Your friend who is helping you has perfect pitch and claims the frequency that you both hear is 280 Hz. You wonder if the truck is speeding. The speed limit is 65 mi/hr. You remember that the speed of sound is about 1100 ft/s which is 750 mph.

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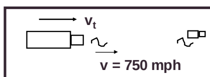


f' = 280 Hz

**Want to find $v_t$**

**We know:**

$f = 1/\tau = 250$ Hz

v = 750 mph

**You hear**

$f' = 1/\tau' = 280$ Hz

speed limit = 65 mph

**Approach:**

Use kinematics

Use the relationship of frequency to wave velocity.

Speed of wave does not depend on the speed of the object emitting it.

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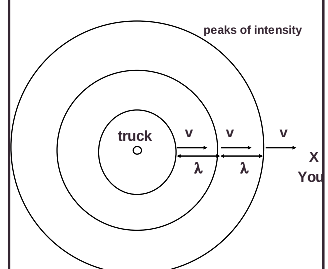
**Diagram of sound wave for a truck at rest.**

Distance between peaks of intensity is the same everywhere



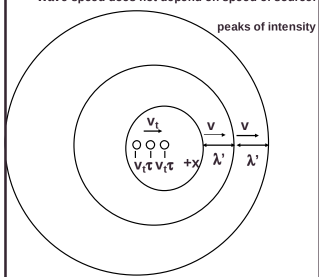

$$\lambda = v\tau$$

$$f = \frac{v}{\lambda}$$

**Diagram of sound waves for truck moving with speed $v_t$**

Distance between peaks of intensity is not the same everywhere.

Wave speed does not depend on speed of source.

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**Wave peaks are closer together in direction truck is moving.**

Truck moves toward wave peaks

If truck is emitting a sound with a period $\tau$

The distance between peaks is

$$\lambda - v_t\tau$$

So the distance between repetitions is

$$\lambda' = \lambda - v_t\tau$$

$\tau$ is time between emitted peaks.

For any wave there is a relationship between wavelength and period

$$\lambda' = v\tau'$$

$\tau'$ is the time between received peaks

---

**Find $v_t$**

$$\lambda - v_t\tau = v\tau'$$

need $\lambda$, $\tau$, $\tau'$

Find $\tau'$

$$f' = \frac{1}{\tau'}$$

Find $\lambda$

$$\lambda = v\tau$$

Find $\tau$

$$f = \frac{1}{\tau}$$

$$\frac{v}{f} - \frac{v_t}{f} = \frac{v}{f'}$$

$$\frac{v}{f} - \frac{v}{f'} = \frac{v_t}{f}$$

$$v - v\frac{f}{f'} = v_t$$

$$v\left(1 - \frac{f}{f'}\right) = v_t$$

check units

---

$$750\,\frac{mi}{hr}\left(1 - \frac{250Hz}{280Hz}\right) = v_t = 80\,\frac{mi}{hr}$$

units are correct, speed in mph

The truck is speeding but 80 mph is possible for a truck

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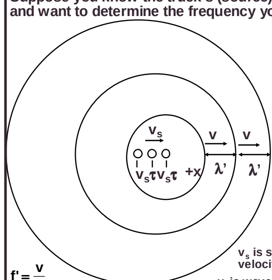
**Many possible variations**

Suppose you know the truck's (source) speed and want to determine the frequency you hear




$v_s$ is source velocity

v is wave velocity

$$f' = \frac{v}{\lambda'}$$

since truck moves toward you a distance $v_t\tau$

$$\lambda' = \lambda - v_s\tau$$

$$\lambda' = \lambda - v_s/f$$

$\tau$ is time between emitted peaks.

know v, $v_s$, and f want f'

---

**Find f'**

$$f' = \frac{v}{\lambda'}$$

Find $\lambda'$

$$\lambda' = \lambda - v_s/f$$

Find $\lambda$

$$\lambda = v/f$$

$$\lambda' = v/f - v_s/f$$

$$f' = \frac{v}{\left(\frac{v}{f} - \frac{v_s}{f}\right)}$$

$$f' = f\frac{v}{(v - v_s)}$$

as $v_s$ gets larger, the frequency heard gets greater.

Higher pitch

If the source is going away from you

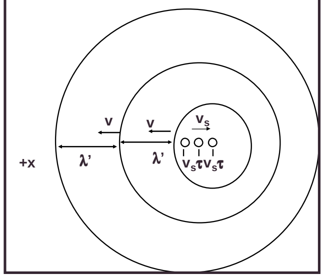
$v_s$ has the opposite sign

$$f' = f\frac{v}{(v+v_s)}$$

as $v_s$ gets larger, the frequency heard gets less.

Lower pitch

For practice draw the diagram and show this is true

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**Example:**

While trying to get home for spring break, you are driving down the freeway. On the side of the road is a truck with a flat tire. Since the driver needs help, he sounds his air horn as you pass him. Your friend who is riding with you has perfect pitch and claims the frequency that you both hear is 280 Hz. You wonder what the frequency of the truck's horn is. You know you are driving right at the speed limit of 65 mph. You remember that the speed of sound is about 1100 ft/s which is 750 mph.

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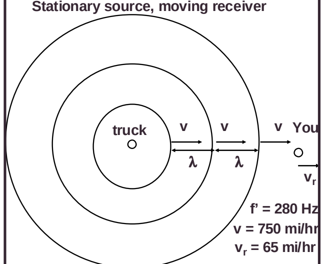
The truck is stopped

Stationary source, moving receiver




$f' = 280$ Hz

$v = 750$ mi/hr

$v_r = 65$ mi/hr

What frequency do you hear?

Speed of wave depends only on the material it travels through. v

$\tau$ is the period of the wave emitted by the source.

$$\lambda = v\tau$$

f is the frequency of the wave emitted by the source.

$$f = \frac{v}{\lambda}$$

---

Since you are moving away from the waves, you observe them to have a slower speed,

$$v' = v - v_r$$

The distance between the wave peaks is the same, so you observe the same $\lambda$

The frequency you hear is greater since the time between peaks is smaller than if you were standing still.

$\lambda = v'\tau'$ $\quad f' = \frac{v'}{\lambda}$

Find f

$f = \frac{v}{\lambda}$ 1 need $\lambda$

Find $\lambda$

$f' = \frac{v'}{\lambda}$

$\lambda = \frac{v'}{f'}$ into 1

$f = f'\frac{v}{v'}$ 2 need v'

Find v'

$v' = v - v_r$ into 2

---

$$f = f'\frac{v}{v - v_r}$$

$$f = (280\text{Hz})\frac{750\frac{mi}{hr}}{\left(750\frac{mi}{hr} - 65\frac{mi}{hr}\right)} = 307\text{Hz}$$

Units are correct, frequency in Hz

Answer is reasonable since the moving receiver hears a lower frequency than emitted by the source.

$f' = f\frac{v - v_r}{v}$ solving for f' from above

If the receiver is moving away from the source

$v' = v - (-v_r) = v + v_r$

replace $v_r$ by $-v_r$

moving source (toward receiver) $f' = f\frac{v}{(v - v_s)}$

moving receiver (toward source) $f' = f\frac{v + v_r}{v}$

---

RECAP

Moving source $\quad \lambda' = v_s\tau$

| Emitted | Observed | |
|---|---|---|
| v | v | |
| $\lambda$ | $\lambda' < \lambda$ source toward receiver | |
| | $\lambda' > \lambda$ source away from receiver | |
| | $\lambda' = \lambda - v_s\tau$ | motion of s |
| $\lambda = v\tau$ | $\lambda' = v\tau'$ | from wave |
| $\lambda f = v$ | $\lambda' f' = v$ | equation |

Moving receiver

| Emitted | Observed | |
|---|---|---|
| v | v' | |
| | v' > v receiver toward source | |
| | v' < v receiver away from source | |
| | $v' = v + v_r$ | motion of r |
| $\lambda$ | $\lambda$ | |
| $\lambda = v\tau$ | $\lambda = v'\tau'$ | from wave |
| $\lambda f = v$ | $\lambda f' = v'$ | equation |

**Both moving source and receiver**

For emitted wave $\lambda f = v$

For the wave you hear $\lambda' f' = v'$

moving source $\lambda' = \lambda - v_s\tau = \lambda - v_s/f$

Source moving in SAME direction as wave

moving receiver $v' = v - v_r$

Receiver moving in SAME direction as wave

Know: $f, v_s, v_r$

Find f'

$\lambda' f' = v'$ need $\lambda', v'$

$$\left(\lambda - \frac{v_s}{f}\right) f' = (v - v_r)$$ need $\lambda$

$$\lambda = v/f$$

$$\left(\lambda - \frac{v_s}{f}\right) f' = (v - v_r)$$

$$(v - v_s)\frac{f'}{f} = (v - v_r)$$

$$\boxed{f' = f\frac{v - v_r}{(v - v_s)}}$$