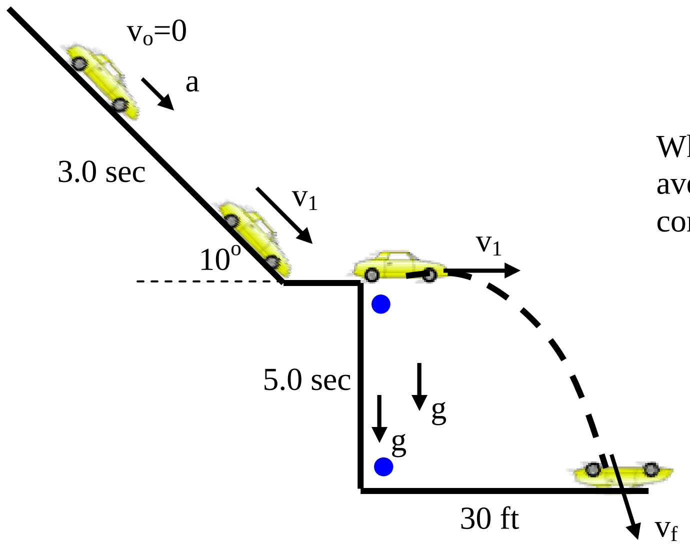
Solutions to quiz 1

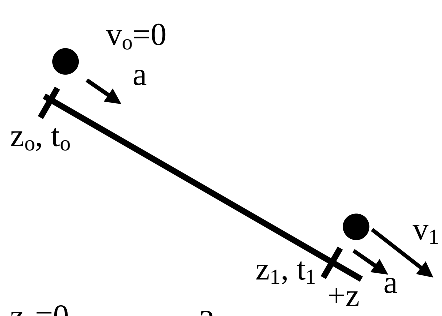
1. Group part.



What is the car's average acceleration coming down the hill?

Approach: Use definition of average acceleration for the car on the hill.
- Perpendicular components of motion are independent
- When car is in the air
  - Horizontal velocity is constant
  - Average horizontal velocity = instantaneous horizontal velocity
  - Vertical acceleration is constant
  - Average horizontal acceleration = instantaneous horizontal acceleration
- Stone and car have the same vertical acceleration and same initial vertical velocity so they fall in the same time.
- Assume car does not slow down in short parking lot.
- Neglect air resistance.

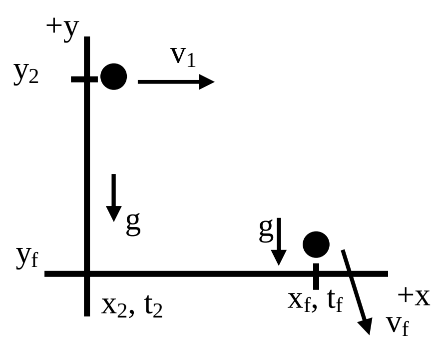
Motion diagram on the hill:



$z_o=0$	$z_1=?$
$t_o=0$	$t_1=3.0$ s
$v_o=0$	$v_1=?$
$a=?$	

Target: a

Motion diagram in the air:



$x_2=0$	$x_f=30$ ft
$y_2=?$	$y_f=0$
$t_2=?$	$t_f-t_2=5.0$ s
$v_1=?$	$v_f=?$
$g=9.8\ m/s^2$	

On the hill: $a_{av} = \frac{v_1 - v_o}{t_1 - t_o} = \frac{v_1}{t_1}$ (definition of average acceleration)

In the air: $v_{avx} = v_x = v_1 = \frac{x_f - x_2}{t_f - t_2} = \frac{x_f}{t_f - t_2}$ (horizontal velocity is constant)

$$a_{avy} = a_y = -g = \frac{-v_{fy} - v_{2y}}{t_f - t_2} = \frac{-v_{fy}}{t_f - t_2}$$ (vertical acceleration is constant)

$$y_f = -\frac{1}{2}g(t_f - t_2)^2 + v_{2y}(t_f - t_2) + y_2$$ (vertical acceleration is constant)

$$0 = -\frac{1}{2}g(t_f - t_2)^2 + y_2$$

Plan:	unknowns
Find a_{av}	a_{av}
$a_{av} = \frac{v_1}{t_1}$ 1	v_1
Find v_1	
$v_1 = \frac{x_f}{t_f - t_2}$ 2	

2 unknowns and 2 equations, can be solved

$v_1 = \frac{x_f}{t_f - t_2}$ put into 1

$$a_{av} = \frac{\frac{x_f}{t_f - t_2}}{t_1} = \frac{x_f}{t_1(t_f - t_2)}$$

check units: $a_{av} = \frac{[m]}{[s][s]} = \frac{[m]}{[s]^2}$ correct for acceleration

$$a_{av} = \frac{30\text{ft}}{3.0\text{s}(5.0\text{s})} = 2.0\frac{\text{ft}}{\text{s}^2}$$

Evaluation:

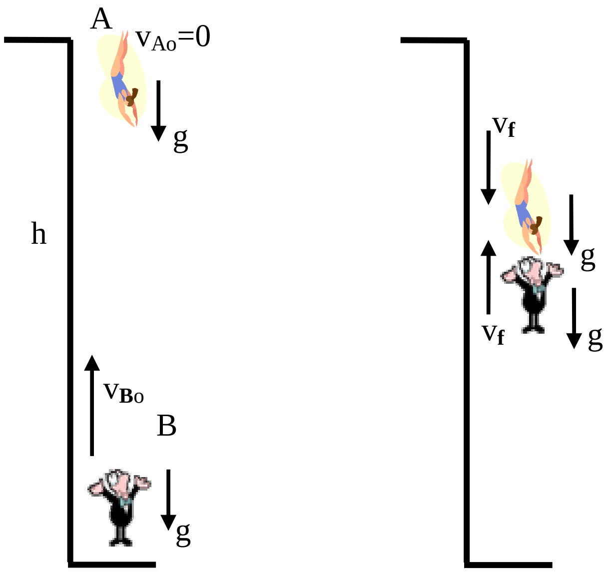
ft/s^2 are the correct units for an acceleration.

The average acceleration down the hill is less than the free fall acceleration which is reasonable.
In the equation for average acceleration, if the car fell further from the hill, the average acceleration would be larger. That makes sense since the car would have been moving faster when it went over the cliff.

The average acceleration of the car on the hill answers the question.

Solutions to Individual Part of Quiz 1

2.



What fraction of the height of the platform does B travel before the catch? Find y_f in terms of h.

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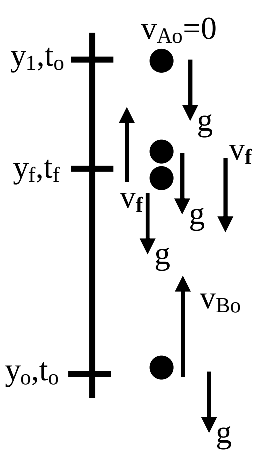
Approach: Use constant acceleration kinematics for both A and B. Acceleration of both people is g. Neglect their interaction with the air. Instantaneous acceleration = average acceleration.

Motion diagram:



$y_1 = h$
$t_o = 0$
$v_{Ao} = 0$
$g = 9.8\ m/s^2$
$y_o = 0$
$v_{Bo} = ?$
$y_f = ?$
$t_F = ?$
$v_f = ?$

Target: y_f

Constant acceleration for A:

$$a = a_{av} = \frac{v_f - v_o}{t_f - t_o} = \frac{-v_f}{t_f} = -g$$

$$y_f = -\frac{1}{2}g(t_f)^2 + h$$

Constant acceleration for B:

$$a = a_{av} = \frac{v_f - v_o}{t_f - t_o} = \frac{v_f - v_{Bo}}{t_f} = -g$$

$$y_f = -\frac{1}{2}g(t_f)^2 + v_{Bo}t_f$$

Plan		unknowns
Find y_f		y_f
$y_f = -\frac{1}{2}g(t_f)^2 + v_{Bo}t_f$	1	t_f , v_{Bo}
Find t_f		
$y_f = -\frac{1}{2}g(t_f)^2 + h$	2	
Find v_{Bo}		
$\frac{v_f - v_{Bo}}{t_f} = -g$	3	v_f
Find v_f		
$\frac{-v_f}{t_f} = -g$	4	

4 unknowns, 4 equations   ok to solve

$$\frac{-v_f}{t_f} = -g$$

$v_f = gt_f$ put into 3

$$\frac{gt_f - v_{Bo}}{t_f} = -g$$

$$gt_f + gt_f = v_{Bo}$$

$2gt_f = v_{Bo}$ put into 1

$$y_f = -\frac{1}{2}g(t_f)^2 + (2gt_f)t_f$$

$y_f = \frac{3}{2}g(t_f)^2$ get t_f^2 from 2

$$y_f = -\frac{1}{2}g(t_f)^2 + h$$

$$\frac{1}{2}g(t_f)^2 = h - y_f$$

$$(t_f)^2 = 2\frac{h - y_f}{g}$$ put into 1

$$y_f = \frac{3}{2}g\left(2\frac{h - y_f}{g}\right)$$

$$y_f = 3(h - y_f)$$

$$4y_f = 3h$$

$$\boxed{\frac{y_f}{h} = \frac{3}{4}}$$

check units: $\frac{[m]}{[m]} = 1$ no units is correct for a fraction.

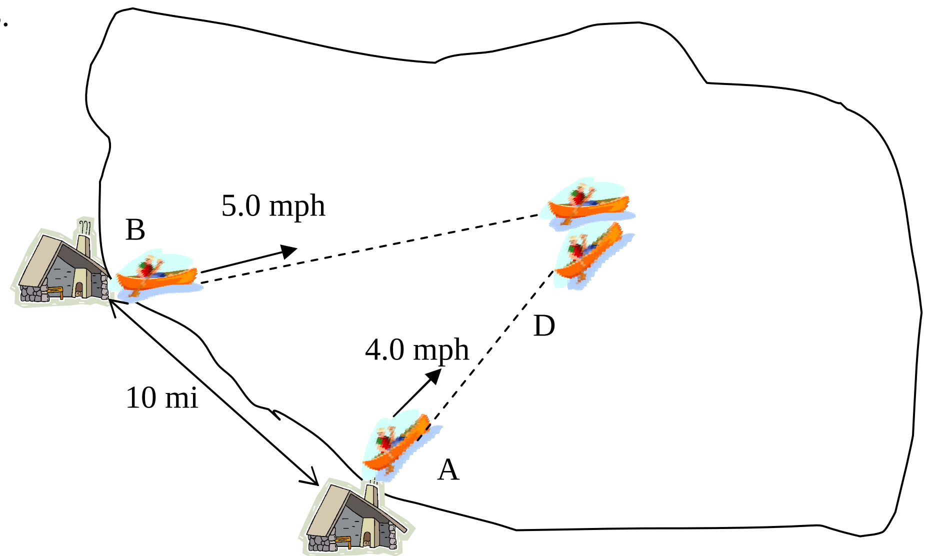
Evaluation:

The answer has no units which is correct for a fraction.

The answer is reasonable since it is between the platform and the trampoline. The fraction is less than 1 and greater than 0.

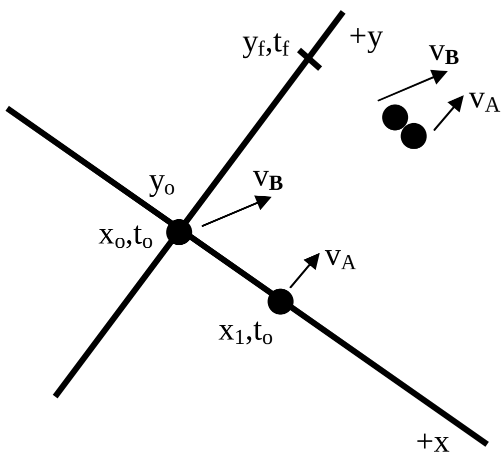
The question has been answered. The fraction of the distance between the trampoline and the platform that the catch takes place is ¾.

3.



What distance does A travel before met by B?

Approach: Both canoes travel with constant velocity so average velocity = instantaneous velocity.
Use the definition of average velocity.
Perpendicular components of velocity are independent.



$x_o = 0$
$t_o = 0$
$y_0 = 0$
$v_B = 5.0$ mi/hr
$x_1 = 10$ mi
$v_A = 4.0$ mi/hr
$y_f = ?$
$t_F = ?$

$$v_A = v_{Aav} = \frac{y_f - y_o}{t_f - t_o} \qquad v_{Bx} = v_{Bxav} = \frac{x_1 - x_o}{t_f - t_o} \qquad v_{By} = v_{Byav} = \frac{y_f - y_o}{t_f - t_o}$$

$$v^2_{Bx} + v^2_{By} = v^2_B$$

Target: y_f

Plan	Unknowns

Find y_f — unknown: y_f

$$v_A = \frac{y_f}{t_f} \qquad 1$$

Unknown: t_f

Find t_g

$$v_{Bx} = \frac{x_1}{t_f} \qquad 2$$

Unknown: v_{Bx}

Find v_{Bx}

$$v_{Bx}^2 + v_{By}^2 = v_B^2 \qquad 3$$

Unknown: v_{By}

Find v_{By}

$$v_{By} = \frac{y_f}{t_f} \qquad 4$$

4 unknowns, 4 equations – ok to solve

$$v_{Bx}^2 + \left(\frac{y_f}{t_f}\right)^2 = v_B^2 \quad \text{solve for } v_{Bx}$$

$$v_{Bx} = \sqrt{v_B^2 - \left(\frac{y_f}{t_f}\right)^2}$$

$$\sqrt{v_B^2 - \left(\frac{y_f}{t_f}\right)^2} = \frac{x_1}{t_f} \quad \text{solve for } t_f$$

$$v_B^2 - \left(\frac{y_f}{t_f}\right)^2 = \left(\frac{x_1}{t_f}\right)^2$$

$$v_B^2 t_f^2 - y_f^2 = x_1^2$$

$$t_f = \sqrt{\frac{x_1^2 + y_f^2}{v_B^2}}$$

$$v_A = \frac{y_f}{\sqrt{\frac{x_1^2 + y_f^2}{v_B^2}}} \quad \text{solve for } y_f$$

$$v_A\sqrt{\frac{x_1^2+y_f^2}{v_B^2}}=y_f$$

$$v_A^2\ \frac{x_1^2+y_f^2}{v_B^2}=y_f^2$$

$$v_A^2 x_1^2+v_A^2 y_f^2=y_f^2 v_B^2$$

$$v_A^2 x_1^2=y_f^2 v_B^2-v_A^2 y_f^2$$

$$v_A^2 x_1^2=y_f^2\left(v_B^2-v_A^2\right)$$

$$\frac{v_A^2 x_1^2}{\left(v_B^2-v_A^2\right)}=y_f^2$$

$$\frac{v_A x_1}{\sqrt{\left(v_B^2-v_A^2\right)}}=y_f$$

check units: $\dfrac{\left[\frac{m}{s}\right][m]}{\sqrt{\left[\frac{m}{s}\right]^2}}=[m]=y_f$ [m] = y_f correct units

$\dfrac{(10\ \text{mi})(4.0\ \text{mi/hr})}{\sqrt{(5.0\ \text{mi/hr})^2-(4.0\ \text{mi/hr})^2}}=y_f=13.3\ \text{mi}$ from the shore along a line perpendicular to the two cabins.

The units (miles) are correct for a distance

13 miles is a reasonable distance to paddle. It is slightly more than the distance that the cabins are apart.

From the equation that represents the answer, $\dfrac{v_A x_1}{\sqrt{\left(v_B^2-v_A^2\right)}}=y_f$, if the second person goes faster (v_B larger), the meeting will take place at a smaller distance (y_f) is smaller. This is reasonable.

The question has been answered, they will meet 13.3 miles from A's cabin perpendicular to the line between the two cabins.

Conceptual Questions:

1. b
2. e
3. d
4. e
5. c
6. c
7. b
8. d
9. d
10. a