This week in Physics

Begin oscillations (Chap. 14)

So far

Interaction of objects to predict their

Center of mass motion

Objects as "point" particles

Rotational motion

Objects as "rigid" bodies

Now to get closer to real objects

Vibrational motion

Objects as oscillators

What do you know about oscillations

Strike something (interaction)

It vibrates

Periodic motion

Back and forth

Up and down

In and out

Structures have a "Natural" frequency

Bell

Guitar string

Spring

Molecules

Electric circuit

Bridge

For concreteness

Think of spring

Experience in your lab

How to describe the oscillation

Size of oscillation

Amplitude

Time for repetition of oscillation

Period

or

Frequency = 1/period

Feature of interaction which yields oscillations

Repeat motion means

non-constant acceleration

(cannot use constant acceleration kinematics!!)

need calculus

Requires non-constant forces

Forces which change with position

Spring force is -kx

Mathematical description of position

Functions that repeat

sin (something)

cos (something)

For example:

y = A sin (something)

Exploring the math of periodic functions

to describe position of oscillating object

Block on end of spring

y = A sin (something)

What is the "A" doing?

Maximum size of sin (something)?

1

"A" needed to give maximum size of oscillation.

Amplitude

What is the "something" doing?

Position varies with time

"something" is a function of time

For sin (something) to make sense

"something cannot have units

for example "something" could be

bt

units of b

1/sec

bt in radians

y = A sin (bt) is a possible function

to describe the position of an oscillating object.

Position repeats

y = A sin (bt +c) would also work

What does this mathematics have to do with reality?

Motion of object attached to spring.

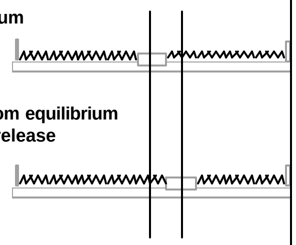
Forces

As in lab determine the motion of the object

Glider on airtrack connected to two springs as an example

At equilibrium

Displace from equilibrium and release



Know:

mass of glider : m_g

spring constant of each spring : k_1 , k_2

Question:

What is position of glider as a function of time?

Use:

dynamics: $\sum F_{horizontal} = m_g\ a_{horizontal}$

Assume:

no friction

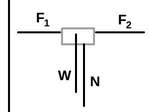
massless springs

Free body diagram of glider

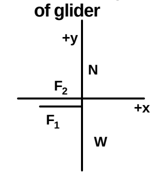
Force diagram of glider

Forces on glider

Free body diagram of glider



Force diagram of glider



$\sum F_x = -F_1 - F_2 = -m_g a_x$

$F_1 = -k_1 x$

$F_2 = -k_2 x$

$k_1 x + k_2 x = -m_g a_x$

Motion of glider

$a_x = \frac{dv_x}{dt}$

$v_x = \frac{dx}{dt}$

$a_x = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$a_x = \frac{d^2x}{dt^2}$

Target: x(t)

$(k_1 + k_2)x = -m_g \frac{d^2x}{dt^2}$

solve for x

Guess and plug it in to check

Method I (physical insight)

I know motion goes back and forth

Periodic

Try a sin (something) function

$x = A \sin(bt + c)$

Method II (math insight)

What function has the same form as its second derivative ?

Try a sin (something) function

$x = A \sin(bt + c)$

$\frac{dx}{dt} = A\cos(bt + c)b$

$\frac{d^2x}{dt^2} = -A\sin(bt + c)b^2$

$(k_1 + k_2)A\sin(bt + c) = -m_g\left(-A\sin(bt + c)b^2\right)$

$(k_1 + k_2)A\sin(bt + c) = m_g\left(A\sin(bt + c)b^2\right)$

Is this equation correct?

ok if

$(k_1 + k_2) = b^2 m_g$

$\frac{(k_1 + k_2)}{m_g} = b^2$

$\sqrt{\frac{(k_1 + k_2)}{m_g}} = b$

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check units

$x = A \sin(bt + c)$

units of b should be 1/s

units of k?

$[F] = [k][x]$

$\left[\frac{kg}{s^2}\right] = [k]$

$\sqrt{\frac{\left[\frac{kg}{s^2}\right]}{[kg]}} = [b]$ ok

$x = A \sin(bt + c)$ is a solution to the force equ.

$\sum F_x = m_g a_x$

$(k_1 + k_2)x = -m_g \frac{d^2x}{dt^2}$

if $\sqrt{\frac{(k_1 + k_2)}{m_g}} = b$

$x = A \sin(bt + c)$

What is the physical meaning of

A

b

c

Since the maximum value of $\sin(bt + c) = 1$

maximum value of $x = A$

A is the maximum displacement of the glider from the equilibrium position.

If its initial displacement is x_o and its initial velocity is 0

$A = x_o$

c tells displacement at t = 0

If t=0 is at time of release of glider from its initial displacement

$x(t=0) = A$

$x = A \sin(bt + c)$

$x_o = A \sin(0 + c)$

$x_o = x_o \sin(c)$

$1 = \sin(c)$

$c = \pi/2$

The value of A and c depend on the specific circumstances of the situation

Initial conditions

A is called Amplitude

c is called phase (usually ϕ)

The value of b depends on the forces

What is the physical significance of b ?

$x = A \sin(bt + \phi)$

x repeats after one period

x at (t=0) = x at (t=T)

x at (t=0) = $A \sin(\phi)$

x at (t=T) = $A \sin(bT + \phi)$

$A \sin(c) = A \sin(bT + \phi)$

$\sin(\phi) = \sin(bT + \phi)$

sin(something) repeats after 2π

$\phi + 2\pi = bT + \phi$

$2\pi = bT$

$b = \frac{2\pi}{T}$

$\frac{1}{T}$ is the frequency

$b = 2\pi f$

b gives the oscillation frequency

$\sqrt{\frac{(k_1 + k_2)}{m_g}} = b$

Depends only on spring constants and mass of glider

Independent of amplitude and phase

Initial conditions

How you started it going

How to solve for the vibrational motion of and object.

1. Write down Newton's second law for that object.

   Useful picture

   Free body and force diagrams

2. Guess the solution to that force equ.

   Three constants

   amplitude, frequency, phase

3. Check to see if guess is satisfies force equation

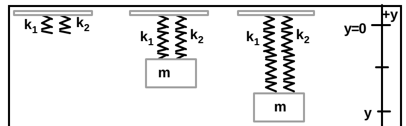
   Gives the frequency

4. Check to see if guess satisfies initial conditions.

   Gives amplitude and phase.

Example:

In your lab, you hang an object from two springs each with a different spring constant. Each spring has one end connected to a stand and the other end connected to the object. Determine how the spring constants are related to the frequency of oscillation of the object.

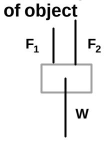


Find the frequency of oscillation of the object

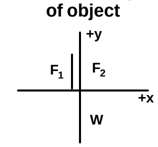
Use dynamics: $\Sigma F_{vertical} = ma_{vertical}$

Ignore friction, mass of springs

Free body diagram of object



Force diagram of object



$\sum F_y = F_1 + F_2 - W = ma_y$

$F_1 = -k_1 y$

$F_2 = -k_2 y$

$-k_1 y - k_2 y - mg = m\frac{d^2y}{dt^2}$

$$-(k_1 + k_2)y - mg = m\frac{d^2y}{dt^2}$$

Solution: $y = A\cos(2\pi ft + \phi) + d$

Target: f

Find f

$$\frac{dy}{dt} = -A\sin(2\pi ft + \phi)2\pi f$$

$$\frac{d^2y}{dt^2} = -A\cos(2\pi ft + \phi)(2\pi f)^2$$

$$-(k_1 + k_2)(A\cos(2\pi ft + \phi) + d) - mg =$$
$$-mA\cos(2\pi ft + \phi)(2\pi f)^2$$

ok if $-(k_1+k_2)d = mg$

$$-(k_1 + k_2)(A\cos(2\pi ft + \phi)) =$$
$$-mA\cos(2\pi ft + \phi)(2\pi f)^2$$

$$-(k_1 + k_2) = -m(2\pi f)^2$$

$$\frac{(k_1 + k_2)}{m} = (2\pi f)^2$$

$$\boxed{\frac{1}{2\pi}\sqrt{\frac{(k_1 + k_2)}{m}} = f}$$

check units:

Compare with frequency for single spring

$k_1 = k$

$k_2 = 0$

$$\frac{1}{2\pi}\sqrt{\frac{k}{m}} = f$$

Equivalent behavior if

$k = k_1 + k_2$