This Week

Continue describing interactions from the viewpoint of CONSERVATION

Conservation of Energy - a scalar

$E_{final} - E_{initial} = E_{input} - E_{output}$

$\Delta E_{system} = \Delta E_{transfer}$

$E_{system} = KE + PE$ $\qquad E_{transfer} = \int_{path} \vec{F} \cdot d\vec{\ell}$

NEW Theory

Conservation of Momentum - a vector

A bit of Chapter 8 - 8.4, 8.6

Competent Problem Solver Chapter 5

Laboratory 5

New Equations

$\vec{p}_{final} - \vec{p}_{initial} = \vec{p}_{input} - \vec{p}_{output}$

$\Delta\vec{p}_{system} = \Delta\vec{p}_{transfer}$

$\vec{p}_{system} = m\vec{v}$ $\qquad \vec{p}_{transfer} = \int_{path} \vec{F} dt$

Conservation Theory

Conservation of Energy

Useful but not complete

No directional information

Energy is a scalar

Need a vector quantity

Property of vector

perpendicular components are independent

Momentum in x direction $\quad p_x = mv_x$

Momentum in y direction $\quad p_y = mv_y$

Momentum in z direction $\quad p_z = mv_z$

Conservation of momentum in each direction is independent

$p_{xfinal} - p_{xinitial} = p_{xinput} - p_{xoutput}$

$p_{yfinal} - p_{yinitial} = p_{yinput} - p_{youtput}$

$p_{zfinal} - p_{zinitial} = p_{zinput} - p_{zoutput}$

Why We Need Momentum

Example:

Your experience in the Lab

Behavior of collisions (interactions)

Take an extreme example to see why conservation of energy is not enough.

Situation: Two objects go towards each other

same mass, same speed

stick together after they hit

Stick together means:

$v_{1f} = v_{2f} = v_f$

initial time $\qquad$ final time

What is v_f?

Conservation of Energy

$E_f - E_i = E_{input} - E_{output}$

system: both objects

initial energy of system

$E_i = (KE)_{1i} + (KE)_{2i}$

final energy of system

$E_f = (KE)_{1f} + (KE)_{2f}$

energy transfer between initial and final time

No external forces in the direction of velocity

Weight and normal force cannot transfer energy to system

Neglect friction

$E_{input} = 0 \qquad E_{output} = 0$

$E_f - E_i = E_{input} - E_{output}$

$(1/2)\, mv_f^2 + (1/2)\, mv_f^2 - (1/2)\, mv_i^2 - (1/2)\, mv_i^2 = 0$

$v_f^2 + v_f^2 - v_i^2 - v_i^2 = 0$

$2\, v_f^2 = 2\, v_i^2$

$v_f = v_i$

or

$v_f = - v_i$

Conservation of energy gives two possibilities!

What does this mean?

Is something wrong with the theory of Conservation of Energy?

Test this prediction

Results

Conservation of Energy for 2 object system predicts

$v_f = v_i \quad$ or $\quad v_f = - v_i$

We observe

$v_f = 0$

Either the theory of conservation of energy is wrong

or

we have not identified all of the terms in the conservation of energy equation

Could there be any energy transfer?

The discrepancy is large

No large interactions with other objects that are not accounted for

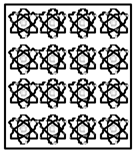
Could there be a system energy different from

Kinetic Energy

Potential Energy

Conservation of Energy

The objects in our system are not really single objects



Internal Structure

Internal parts have energy

Kinetic Energy

Potential Energy

Energy of the system of two objects =

$KE_1 + KE_2 + PE_{12}$

$+ \text{internal } KE_1 + \text{internal } PE_1$

$+ \text{internal } KE_2 + \text{internal } PE_2$

Conservation of Energy

$E_f - E_i = E_{input} - E_{output}$

$E_i = KE_i + PE_i + IE_i$

$E_f = KE_f + PE_f + IE_f$

To use conservation of energy you must know how the internal energy of your system changes

A Vector Theory

Suppose an interaction changes the internal energy of a system

For example a collision that increases the motion of the molecules

Velocity of the molecules is in random directions

Energy is a scaler

It all adds up

If the quantity conserved were a vector

Velocity in opposite directions would cancel

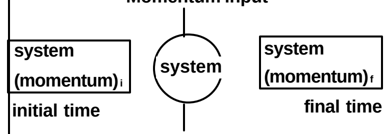
No effect of random internal motion changes

Invent a new vector quantity to conserve

Test to see if it describes the real world

Another Conservation Theory

Conservation of Momentum



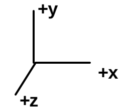
$\bar{p}_f - \bar{p}_i = \bar{p}_{input} - \bar{p}_{output}$

$\Delta \bar{p}_{system} = \Delta \bar{p}_{transfer}$

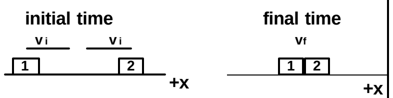
$p_{xf} - p_{xi} = p_{xinput} - p_{xoutput}$

$p_{yf} - p_{yi} = p_{yinput} - p_{youtput}$

$p_{zf} - p_{zi} = p_{zinput} - p_{zoutput}$



Apply Conservation of Momentum to This Interaction



System: both objects

Initial system momentum is the sum of the momentum of each object before the collision

$p_{ix} = mv_{ix} + m(-v_{ix})$

Final system momentum is the sum of the momentum of each object after the collision

$p_{fx} = mv_{fx} + mv_{fx} = 2mv_{fx}$

No external interactions in x direction. Interactions with air (friction and sound can be ignored)

$p_{transferx} = 0$

Conservation of Momentum

$p_{xf} - p_{xi} = p_{xinput} - p_{xoutput}$

$2mv_{fx} - (mv_{ix} - mv_{ix}) = 0$

$2mv_{fx} = 0$

$v_{fx} = 0$

This is exactly what we observe !!

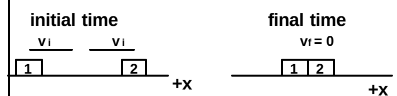
There were no internal momentum changes

Random velocity of molecules gives

$\Sigma \bar{p}_{internal} = 0$

Calculate the internal energy change

Internal Energy Change



Conservation of energy $E_f - E_i = E_{input} - E_{output}$

System both objects

$E_i = KE_i + IE_i$ $E_f = KE_f + IE_f$ $E_{transfer} = 0$

$$\frac{1}{2}(2m)v_f^2 + IE_f - \left(\frac{1}{2}(m)v_i^2 + IE_i\right) = 0$$

$$IE_f - \left(\frac{1}{2}(m)v_i^2 + IE_i\right) = 0$$

$$IE_f - IE_i = \frac{1}{2}(m)v_i^2$$

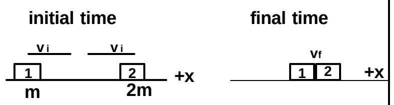
The change of the internal energy (ΔE_i)equals all of the initial kinetic energy

Large change of internal energy

Another Example

What if $m_1 = 2\,m_2$ when objects stick together

How is v_f related to v_i?



$p_{ix} = mv_{ix} - 2mv_{ix}$

$p_{ix} = -mv_{ix}$

$p_{fx} = mv_{fx} + 2mv_{fx}$

$p_{fx} = 3mv_{fx}$

Conservation of Momentum

System: both objects

$p_{xf} - p_{xi} = p_{xinput} - p_{xoutput}$

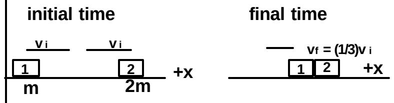
$p_{transfer} = 0$

$3mv_{fx} - (-mv_{ix}) = 0$

$3mv_{fx} = (-mv_{ix})$

$v_{fx} = -\frac{1}{3} v_{ix}$

Internal Energy Change



Conservation of energy $E_f - E_i = E_{input} - E_{output}$

System both objects

$E_i = KE_i + IE_i$ $E_f = KE_f + IE_f$ $E_{transfer} = 0$

$$\frac{1}{2}(3m)\left(\frac{1}{3}v_i\right)^2 + IE_f - \left(\frac{1}{2}(3m)v_i^2 + IE_i\right) = 0$$

$$IE_f - IE_i = \frac{1}{2}(3m)v_i^2 - \frac{1}{2}(3m)\frac{1}{9}v_i^2$$

$$IE_f - IE_i = \frac{8}{9}\left(\frac{1}{2}(3m)v_i^2\right)$$

The change of the internal energy (ΔE_i) equals 89% (8/9) of the initial kinetic energy

Large change of internal energy when objects stick together

Conservation Theories

Two conservation theories to explain everything (so far).

Conservation of Energy

$E_f - E_i = E_{input} - E_{output}$

$\Delta E_{system} = \Delta E_{transfer}$

E_{system} (E_i and E_f) = KE + PE + IE

$E_{transfer}$ (E_{input} and E_{output}) = $\int_{path} \vec{F}\cdot d\vec{l}$

Energy is a scalar (no direction)

Conservation of Momentum

$\vec{p}_f - \vec{p}_i = \vec{p}_{input} - \vec{p}_{output}$

$\Delta\vec{p}_{system} = \Delta\vec{p}_{transfer}$

means

$p_{fx} - p_{ix} = p_{transfer\,x}$

$p_{fy} - p_{iy} = p_{transfer\,y}$

$p_{fz} - p_{iz} = p_{transfer\,z}$

$p_{systemx}$ (p_{ix} and p_{fx}) = Σ m v_x

$p_{transfer\,x}$ (p_{inputx} and $p_{outputx}$) = ?

Momentum is a vector (has direction)

Momentum Transfer

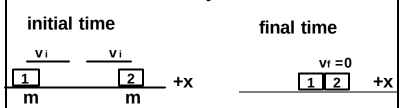
How can we know the momentum transferred to or from a system? $\vec{p}_{transfer}$

The system interacts with external objects.

There must be an external force on the system.

Want a theory of the relationship between the momentum transfer and the external force

Think about a situation you know



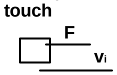
If system is object 1, momentum is transferred from it

$\vec{p}_{output}$

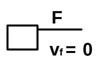
system: single object (1)

one external force - during collision exerted by other object (2)

Initial time: Just as the objects first touch



Final time: Just as the objects stop



Force is probably not constant

How does the change of momentum depend on the force?

$\vec{F} = m\vec{a}$

$\vec{a} = \frac{d\vec{v}}{dt}$ a is not a constant

$\vec{F} = m\frac{d(\vec{v})}{dt}$

$\vec{F} = \frac{d(m\vec{v})}{dt}$ m is a constant

$\vec{F} = \frac{d\vec{p}}{dt}$

$$\int_{beginning}^{end}\vec{F}\,dt = \int_{beginning}^{end}\frac{d\vec{p}}{dt}dt$$

$$\int_{beginning}^{end}\vec{F}\,dt = \vec{p}\Big|_{beginning}^{end} = \vec{p}_f - \vec{p}_i$$

$$\int_{beginning}^{end}\vec{F}\,dt = \vec{p}_f - \vec{p}_i = \Delta\vec{p}_{system}$$

Conservation of momentum:

$\Delta\vec{p}_{system} = \Delta\vec{p}_{transfer}$

$\vec{p}_f - \vec{p}_i = \vec{p}_{output}$

Since in this case:

$\int_{beginning}^{end}\vec{F}\,dt = \Delta\vec{p}_{system}$

$\int_{beginning}^{end}\vec{F}\,dt = \vec{p}_{output}$

Momentum is transferred to a system by an external force on that system acting over a time interval

Direction of momentum transfer is in the direction of that force.

Because momentum is a vector

An external force in one direction (y) does not effect the momentum in a perpendicular direction (x).

$\int_{beginning}^{end} F_x\,dt = p_{x\ transfer}$

$\int_{beginning}^{end} F_y\,dt = p_{y\ transfer}$

Compare Momentum Transfer and Energy Transfer

Momentum Transfer

$$\int_{beginning}^{end} \vec{F}\,dt = \vec{p}_{transfer}$$

Momentum is transferred to a system by

an external force on that system acting over time

All external forces on a system transfer momentum to or from it.

Energy Transfer

$$\int_{path} \vec{F} \cdot d\vec{\ell} = E_{transfer}$$

Energy is transferred to a system by

an external force on that system acting over a distance

An external force perpendicular to the velocity does not transfer energy.

Types of Interactions

Internal Energy Does not Change

Called an ELASTIC interaction

- Very rare
- Mostly an approximation
  - How good?
  - Your laboratory measurements

Both conservation of energy and conservation of momentum are useful

Internal Energy Changes

Called an INELASTIC interaction

Usually only conservation of momentum is useful

Internal energy change shows up as

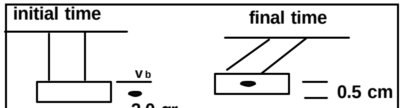
- Temperature change
- Shape change
- Form change

Largest internal energy change if objects stick together - PERFECTLY INELASTIC interaction

Example

To determine the muzzle velocity of a bullet fired by a rifle, you shoot the 2.0 gm bullet into a 2.0 kg wooden block. The block is suspended by wires from the ceiling and it initially at rest. After the bullet is embedded in the block, the block swings up to a maximum height 0.5 cm above its initial position.

initial time final time



Question: What is the speed of the bullet when fired?

Approach:

Try conservation of energy:

System: bullet, block, Earth

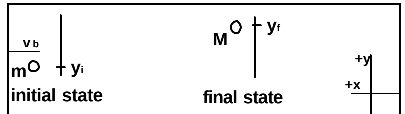
System energy: KE, GPE, IE

Initial time: just before bullet enters block

Final time: block at highest point

External forces on system

Force of wires - perpendicular to path
No energy transfer



initial state final state

$y_i = 0$ $y_f = 0.50$ cm

m = 2.0 gr. M = 2.0 kg + 2.0 gr

System: bullet, block, Earth

$E_f - E_i = E_{input} - E_{output}$

$E_i = KE_i + PE_i + IE_i$

$E_f = KE_f + PE_f + IE_f$

Since the bullet and block stick together

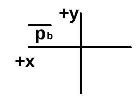
Perfectly inelastic collision
Large internal energy change

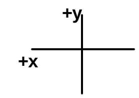
Don't know how the internal energy changes

Conservation of Energy is true but useless

New approach Conservation of momentum

System: bullet, block





initial time final time

$p_{ix} = p_{ix\,bullet} + p_{ix\,block}$ $p_{fx} = p_{fx\,bullet} + p_{fx\,block}$

$p_{ix} = m\,v_b$ $p_{fx} = 0$

conservation of momentum

$$\vec{p}_f - \vec{p}_i = \vec{p}_{input} - \vec{p}_{output}$$

There are external forces acting over time

$$p_{x\ transfer} = \int_{time\ at\ bottom}^{time\ at\ top} T_x\,dt$$

Don't know time of swing or T

Conservation of momentum is true but useless

Neither conservation of energy
nor
conservation of momentum is useful

For the entire event

- Wrong initial time
- Wrong final time
- Wrong system

Need to make some decisions

Is conservation of energy useful for part of the event?

What time interval? What system?

the internal energy of the system is not changing

there is no energy transfer

Is conservation of momentum useful for part of the event?

What time interval? What system?

there is no momentum transfer

Approach

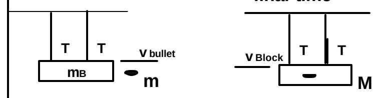
Break problem into time intervals over which each conservation theory is useful

Time interval when conservation of momentum is most useful

From just before the collision (bullet has not entered the block)

to just after the collision (bullet is not moving with respect to the block but the block has not yet moved)

system: bullet, block



$p_{xi} = m_{bullet} v_{bullet}$ $p_{xf} = (m_{block} + m_{bullet}) v_{block}$

Conservation of momentum:

$(m_{block} + m_{bullet}) v_{block} - m_{bullet} v_{bullet} = 0$

Target: v_{bullet}

Time interval when conservation of energy is most useful

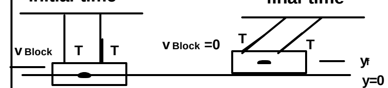
From just after the collision (bullet is not moving with respect to the block)

to when the block (with the bullet in it) reaches its highest point.

system: bullet, block, Earth

No change of internal energy
No energy transfer

Conservation of Energy



$E_i = \frac{1}{2}(m_{block} + m_{bullet}) v_{block}^2$ $E_f = (m_{block} + m_{bullet}) g y_f$

Conservation of energy:

$(m_{block} + m_{bullet}) g y_f - \frac{1}{2}(m_{block} + m_{bullet}) v_{block}^2$

Plan | unknowns

Find v_{bullet} | v_{bullet}

$(m_{block} + m_{bullet}) v_{block} - m_{bullet} v_{bullet} = 0$ | v_{Block}

Find v_{Block}

$(m_{block} + m_{bullet}) g y_f - \frac{1}{2}(m_{block} + m_{bullet}) v_{block}^2$

2 equations, 2 unknowns.....can solve it!

$$g y_f - \frac{1}{2} v_{Block}^2 = 0$$

$$2(g y_f) = v_{Block}^2$$

$$\sqrt{2 g y_f} = v_{Block}$$

$$(m_{block} + m_{bullet})\sqrt{2 g y_f} - m_{bullet} v_{bullet} = 0$$

$$\frac{(m_{block} + m_{bullet})\sqrt{2 g y_f}}{m_{bullet}} = v_{bullet}$$

$$\frac{(m_{block} + m_{bullet})\sqrt{2 g y_f}}{m_{bullet}} = v_{bullet}$$

Any algebra mistakes?

check units

Since we have mass units in numerator and denominator

$$\sqrt{\left[\frac{m}{s^2}\right][m]} = \sqrt{\left[\frac{m^2}{s^2}\right]} = \left[\frac{m}{s}\right]$$ ok units of v

$$\frac{(2.002\,kg)\sqrt{2\left(9.8\frac{m}{s^2}\right)(0.005m)}}{0.002\,kg} = v_{bullet}$$

v_{bullet} = 310 m/s

Check

Question Answered? the speed of the bullet leaving the gun is the same as its speed just before it enters the block.

The units of speed are correct (m/s)

Reasonable?

Bullets travel at about the speed of sound

speed of sound approximately 1100 ft /s

3 ft is about 1 m

v_{bullet} = 930 ft / s

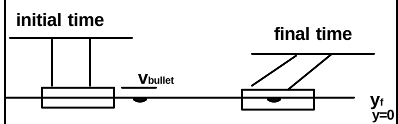
seems to be about as expected

Now we could find the change in internal energy of the system (probably signaled by a rise in temperature of the bullet and block).

Use Conservation of Energy

$E_f - E_i = E_{input} - E_{output}$

System: bullet, block, Earth



We now know

- •KE of system before bullet enters block
- •KE of system when block swings to maximum height
- •Change in PE over this time interval

$E_i = KE_i + PE_i + IE_i$ $\quad E_{input} = 0$

$E_f = KE_f + PE_f + IE_f$ $\quad E_{output} = 0$

$$(m_{block} + m_{bullet}) g y_f + IE_f - \left(\frac{1}{2} m_{bullet} v_{bullet}^2 + IE_i\right) = 0$$

$$(m_{block} + m_{bullet}) g y_f + IE_f - \left(\frac{1}{2} m_{bullet} v_{bullet}^2 + IE_i\right) = 0$$

$$IE_f - IE_i = \frac{1}{2} m_{bullet} v_{bullet}^2 - (m_{block} + m_{bullet}) g y_f$$

$$\Delta IE = \frac{1}{2} m_{bullet} v_{bullet}^2 - (m_{block} + m_{bullet}) g y_f$$

$\Delta IE = (1/2)\ (.002\ kg)(310\ m/s)^2$
$\quad - (2.002\ kg)\ (9.8\ m/s^2)\ (.005\ m)$

$\Delta IE = 96\ kg\ m^2/s^2 - 0.10\ kg\ m^2/s^2$

Note that $\Delta KE >> \Delta PE$

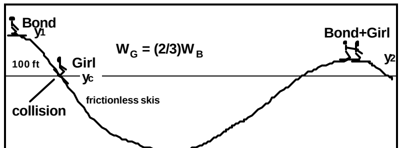
Almost all of the bullet's original KE goes into internal energy of the system

$\Delta IE = 95.9\ m\ kg\ m/s^2 = 95.9\ m\ N = 95.9\ J$

To what height would this energy lift the block (2 kg)?

Example

Because of your physics background, you have been chosen as a technical advisor for a new James Bond movie. In this scene, Bond and his partner, who is 2/3 his weight (including skis, boots, clothes, and various hidden weapons), are skiing in the Alps. She goes on ahead down a slope while he stays at the top to adjust his boot. After skiing down a vertical distance of 100 ft, she stops to wait for him and is captured by the bad guys. Bond notices that she is standing with her skis pointed downhill resting on her poles. To make as little noise as possible, he starts from rest and glides down the slope right at her. He grabs her and they both continue gliding downhill together. At the bottom of the hill another slope goes uphill and they continue gliding, hand-in-hand, up that slope until they reach the top of another hill and are safe. What is the maximum possible height that the second hill? Both Bond and his partner are using new frictionless stealth skis developed for the British Secret Service by Q .



Question:
What is the maximum height of girl+Bond?

Approach:

Inelastic collision - internal energy of girl+Bond changes during the collision

Don't use conservation of energy from beginning to end. (Don't know the change of internal energy)

Don't use conservation of momentum from beginning to end. (Don't know momentum transfer)

Use conservation of energy before collision
frictionless skis -- no energy transfer
Bond, Earth system

Use conservation of momentum from just before collision to just after collision.

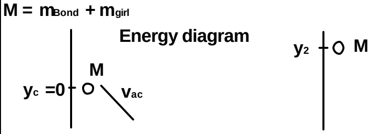
Use conservation of energy after collision

Target quantity: y_2

Time interval: Just after collision to top of hill
System: girl, Bond, Earth

$M = m_{Bond} + m_{girl}$

Energy diagram



$E_f - E_i = E_{transfer}$

$E_{transfer} = 0$

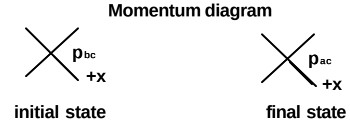
$E_i = (1/2)\ M\ v_{ac}^2$ $\quad E_f = Mgy_2$

Need to know v_{ac}

Time interval: Just before collision to just after collision

System: girl, Bond

Momentum diagram



$p_{fx} - p_{ix} = p_{transfer\ x}$

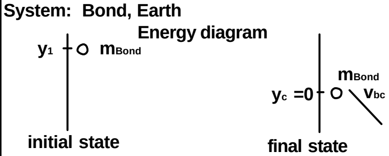
$p_{ix} = m_{bond}\ v_{bc}$ $\quad p_{transfer\ x} = 0$ $\quad p_{fx} = M\ v_{ac}$

need v_{bc}

Time interval: Just before collision to just as Bond starts gliding

System: Bond, Earth

Energy diagram



$E_f - E_i = E_{transfer}$ $\quad E_f = (1/2)\ m_{Bond} v_{bc}^2$

$E_i = m_{Bond}\ g y_1$ $\quad E_{transfer} = 0$

Can not easily use conservation of energy or conservation of momentum from beginning situation to end situation.

Break problem into 3 time intervals

Time interval I

Conservation of energy: system Bond + Earth

$E_i = PE_B = m_B\, g\, y_1$ $E_{transfer} = 0$

$E_f = KE_B = (1/2)\, m_B\, v_B^2$ $\Delta IE = 0$

$(1/2)\, m_B\, v_B^2 - m_B\, g\, y_1 = 0$

$\boxed{v_B^2 = 2\, g\, y_1}$

Time interval II

Cons. of momentum: system Bond+ girl

$p_i = m_B\, v_B$

$p_f = (m_B + m_G)\, v_c$ $p_{transfer} = 0$

$(m_B + m_G)\, v_c - m_B\, v_B = 0$

$v_c = v_B\, [m_B / (m_B + m_c)]$

since $W_G = (2/3)\, W_B$

$m_G = (2/3)\, m_B$

$\boxed{v_c = (3/5)\, v_B}$

Time interval III

Cons. of energy: system Bond+girl+Earth

$E_i = KE_{(B+G)} = (1/2)\, (m_B + m_G)\, v_c^2$

$E_f = PE_{(B+G)} = (m_B + m_G)\, g\, y_2$

$E_{transfer} = 0$ $\Delta IE = 0$

$(m_B + m_G)\, g\, y_2 - (1/2)\, (m_B + m_G)\, v_c^2 = 0$

$\boxed{y_2 = [1/(2g)]\, v_c^2}$

Find y_2

From time interval III

$\boxed{y_2 = [1/(2g)]\, v_c^2}$

Find y_c

From time interval II

$\boxed{v_c = (3/5)\, v_B}$

Find y_B

From time interval I

$\boxed{v_B^2 = 2\, g\, y_1}$

$y_2 = [1/(2g)]\, (9/25) 2\, g\, y_1$

$\boxed{y_2 = (9/25)\, y_1}$ It is less than $(3/5)\, y_1$

$\boxed{y_2 = 36 \text{ ft}}$