Theory of Motion

Definition of velocity and acceleration

Objects accelerate only if they interact with other objects

Perpendicular components of the motion of an object are independent

The horizontal part of the acceleration NEVER affects the vertical part of the velocity

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This theory predicts the motion of all objects.

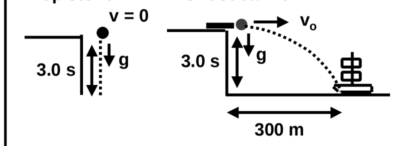
You have tested it in the lab

- Straight line (1-D) motion
- Projectiles (2-D)
- Circular (2-D)

Example

While on a vacation you find an old fort probably built in the 16th century. Large stone walls rise vertically from the shore to protect the fort from cannon fire from pirate ships. Walking around on the ramparts, you find the fort's cannons mounted such that they fire horizontally out of holes near the top of the walls facing the ocean. Leaning out of one of these gun holes, you drop a rock which hits the ocean 3.0 seconds later. You wonder how close a pirate ship would have to sail to the fort to be in range of the fort's cannon? Of course you realize that the range depends on the velocity that the cannonball leaves the cannon. That muzzle velocity depends, in turn, on how much gunpowder was loaded into the cannon. For fun, you decide to calculate the muzzle velocity necessary to hit a pirate ship 300 meters from the base of the fort.

Focus



What is initial velocity of cannon ball?

Use kinematics.

Horizontal motion independent of vertical motion.

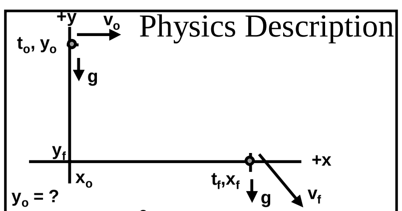
Horizontal: constant velocity

Vertical: constant acceleration

Time for stone to fall = time for cannon ball to fall since gravitational acceleration of falling objects does not depend on mass.

Ignore air resistance

Physics Description



y_o = ?
x_o = 0
t_o = 0
v_o = ?

y_f = 0
x_f = 300 m
t_f = 3.0 s

$v_{fx} = v_o$
v_{fy} = ?
g = 9.8 m/s²

Target: v_o

Horizontal : constant velocity

$v_x = \frac{x_f - x_o}{t_f - t_o}$ $\quad v_o = \frac{x_f}{t_f}$

Vertical: constant acceleration

$a_y = \frac{v_{fy} - v_{oy}}{t_f - t_o}$ $\quad -g = \frac{-v_{fy}}{t_f}$

$y_f = \frac{1}{2}a_y(t_f - t_o)^2 + v_{oy}(t_f - t_o) + y_o$

$0 = -\frac{1}{2}gt_f^2 + y_o$

Plan

unknowns

Find v_o $\quad$ v_o

$v_o = \frac{x_f}{t_f}$

No additional unknowns ! We're done.

Execute the plan

$v_o = \frac{300m}{3.0s} = 100\frac{m}{s}$

Evaluate

Units are correct for a velocity

Speed of a fast car is about 100 km/hr

$100\frac{km}{hr}\left(\frac{1000m}{km}\right)\left(\frac{hr}{3600s}\right) = 28\frac{m}{s}$

so cannon ball is faster than a car

Speed of a jet is about 1000 km/hr = 280 m/s

so cannon ball is slower than a jet

Cannon ball speed is not unreasonable.