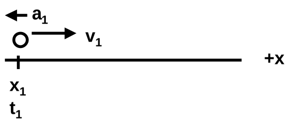
Review

Describe straight line motion

Instantaneous acceleration and instantaneous velocity at a point in space and and instant of time



Average between t_o and t_f

$\bar{v} = \frac{x_f - x_o}{t_f - t_o}$ $\bar{a} = \frac{v_f - v_o}{t_f - t_o}$

Instantaneous at t_1

$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ $a = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$

Instantaneous velocity $v = \left(\frac{dx}{dt}\right)$

ONLY if velocity is constant

Instantaneous velocity = Average velocity

$$v = \frac{x_f - x_o}{t_f - t_o}$$

Instantaneous acceleration $a = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

ONLY if acceleration is constant

Instantaneous accel. = Average accel.

$$a = \frac{v_f - v_o}{t_f - t_o}$$

and

$$x = \frac{1}{2}a_x(\Delta t)^2 + v_{ox}(\Delta t) + x_o$$

but

v is NOT $\frac{x_f - x_o}{t_f - t_o}$

This Week

Application Examples 1 D motion

2 Dimensional Motion

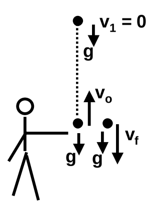
Vectors

Quiz 1

Thursday
Friday

Example

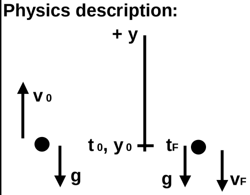
Suppose you throw an ball up and then catch it at the same height you threw it from, how fast does it hit your hand ?



Approach:

Assume we know v_o
- use the definition of velocity
- use the definition of acceleration
- straight line motion
- constant acceleration
  - neglect air resistance

Physics description:



$y_0 = 0$
$t_0 = 0$
$t_f = ?$
$y_F = y_0$
$a = g$

Relevant Equations:

$v = \frac{dy}{dt}$ $a = \frac{dv}{dt}$

acceleration of the ball is constant

$a = \bar{a}$ $\bar{a} = \frac{\Delta v}{\Delta t}$ $-g = \frac{-v_f - v_o}{t_f - t_o}$

$$y_f = \frac{1}{2}a(t_f - t_o)^2 + v_0(t_f - t_o) + y_0$$

$$0 = \frac{1}{2}(-g)t_f^2 + v_0 t_f$$

Target: v_f

Plan:

unknown

Find v_F v_F

$$g = \frac{v_f + v_o}{t_f - t_o}$$

$g = \frac{v_f + v_o}{t_f}$ **[1]** t_F

Find t_F

$$0 = \frac{1}{2}(-g)t_f^2 + v_0 t_f$$

$0 = \frac{1}{2}(-g)t_f + v_0$ **[2]**

2 unknown and 2 equations

Can be solved

[2] $0 = \frac{1}{2}(-g)t_f + v_0$

$\frac{2v_0}{g} = t_F$

Into [1]

$g = \frac{v_f + v_o}{\frac{2v_0}{g}}$

$g = (v_f + v_o)\frac{g}{2v_0}$

$2v_0 = (v_f + v_o)$

$v_0 = v_f$

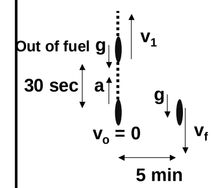
The magnitude of the velocity of the ball hitting your hand is the same as that leaving your hand

he instantaneous velocity of the ball just efore it hits your hand is how fast it is going.

Example

You have a job working for a University research group investigating ozone depletion in the atmosphere. The plan is to collect data on the chemical composition of the atmosphere as a function of the distance from the ground using a mass spectrometer located in the nose cone of a rocket fired vertically. To make sure the delicate instruments survive the launch, your task is to determine the acceleration of the rocket before it uses up its fuel. The rocket is launched straight up with a constant acceleration until the fuel is gone 30 seconds later. To collect enough data, the total flight time must be 5.0 minutes before the rocket crashes into the ground.

Focus



What is the rocket acceleration when it has fuel?

Approach

Use constant accel. kinematics for initial 30 s

Acceleration is up

Average accel = instantaneous accel.

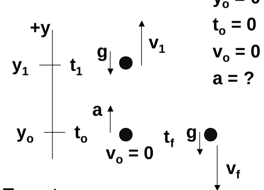
Use constant accel. kinematics after fuel is gone

Acceleration is down = g

Average accel = instantaneous accel.

Neglect air resistance

Motion Diagram



$y_o = 0$ $y_1 = ?$

$t_o = 0$ $t_1 = 30$ s

$v_o = 0$ $v_1 = ?$

$a = ?$ $g = 9.8$ m/s²

$y_f = y_o$

$t_f = 5$ min

$v_f = ?$

Target: a

Relevant equations:

Constant acceleration t_o to t_1

$a = \frac{v_1 - v_o}{t_1 - t_o} = \frac{v_1}{t_1}$ Average accel. = instantaneous accel.

$y_1 = \frac{1}{2}a(t_1 - t_o)^2 + v_o(t_1 - t_o) + y_o = \frac{1}{2}at_1^2$

Constant acceleration t_1 to t_f

$-g = \frac{-v_f - v_1}{t_f - t_1}$ Average accel. = instantaneous accel.

$y_f = 0 = -\frac{1}{2}g(t_f - t_1)^2 + v_1(t_f - t_1) + y_1$

Plan

		unknown
Find a		a
$a = \frac{v_1}{t_1}$	[1]	v_1
Find v_1		
$g = \frac{v_f + v_1}{t_f - t_1}$	[2]	v_f
Find v_f		
Dead end		

We don't have another equation for v_f

We need another way of finding v_1

Plan

		unknown
Find a		a
$a = \frac{v_1}{t_1}$	[1]	v_1
Find v_1		
$0 = -\frac{1}{2}g(t_f - t_1)^2 + v_1(t_f - t_1) + y_1$	[2]	y_1
Find y_1		
$y_1 = \frac{1}{2}at_1^2$	[3]	

3 unknowns, 3 equations

Can be solved

Execute:

[3] $y_1 = \frac{1}{2}at_1^2$ Into [2]

$$0 = -\frac{1}{2}g(t_f - t_1)^2 + v_1(t_f - t_1) + \frac{1}{2}at_1^2$$

$$\frac{1}{2}g(t_f - t_1)^2 = v_1(t_f - t_1) + \frac{1}{2}at_1^2$$

$$\frac{1}{2}g(t_f - t_1)^2 - \frac{1}{2}at_1^2 = v_1(t_f - t_1)$$

$$\frac{\frac{1}{2}g(t_f - t_1)^2 - \frac{1}{2}at_1^2}{(t_f - t_1)} = v_1$$

Into [1]

$$a = \frac{\frac{\frac{1}{2}g(t_f - t_1)^2 - \frac{1}{2}at_1^2}{(t_f - t_1)}}{t_1}$$

$$a = \frac{g(t_f - t_1)^2 - at_1^2}{2t_1(t_f - t_1)}$$

$$a = \frac{g(t_f - t_1)^2 - at_1^2}{2t_1(t_f - t_1)}$$

$$a2t_1(t_f - t_1) = g(t_f - t_1)^2 - at_1^2$$

$$a2t_1(t_f - t_1) + at_1^2 = g(t_f - t_1)^2$$

$$a\left[2t_1(t_f - t_1) + t_1^2\right] = g(t_f - t_1)^2$$

$$a = \frac{g(t_f - t_1)^2}{2t_1(t_f - t_1) + t_1^2}$$

check units

$$[a] = \frac{[a][t^2]}{[t^2] - [t^2]}$$ = units of accel.

OK

$$a = \frac{g(t_f - t_1)^2}{2t_1(t_f - t_1) + t_1^2}$$

$$a = \frac{\left(9.8\frac{m}{s^2}\right)(300s - 30s)^2}{2(30s)(300s - 30s) + (30s)^2}$$

$$a = \frac{\left(9.8\frac{m}{s^2}\right)(300s - 30s)^2}{2(30s)(300s - 30s) + (30s)^2} = 41.8\frac{m}{s^2}$$

Evaluation:

Units are correct for an acceleration

The acceleration is 4.2 g (not unreasonable for a rocket)

This acceleration is what is asked for

Example

A private company suddenly announces that it sent 3 astronauts to Mars and broadcasts TV pictures of the astronauts exploring Mar's surface. Some members of Congress are skeptical and accuse the company of deception. They believe that the TV pictures were faked from a studio on Earth. Because of your knowledge of Physics, you have been hired as part of an investigation team. You notice that the TV recording shows an astronaut dropping an object from the top of a cliff. From the splash of dust visible when the object hits the bottom, you measure that it took 3.0 seconds to hit the ground. Next the astronaut throws another object straight up and it takes 4.0 seconds to hit the bottom of the cliff. From the pictures you determine that this object reached a height of 1.5 meters above the point where both objects were released.

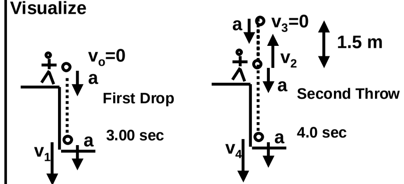
Step 1: Focus the problem

Visualize

Question:

Approach:

Step 1: Focus the problem
Visualize



Question:

What is the acceleration of the object and how does it compare to g?

Approach:

Assume the acceleration of objects is constant on Mars as it is on Earth

Apply kinematics with constant acceleration

Initial time - just after object leaves hand

Final time - just before object hits ground

Ignore air resistance

Step 2 : Describe the Physics

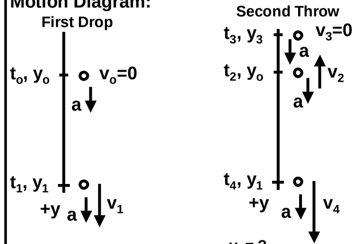
Motion Diagram:

Target Quantity:

Quantitative Relationships:

Step 2 : Describe the Physics

Motion Diagram:



$t_o = 0$, $t_1 = 3.0$ s, $y_o = 0$, $y_1 = ?$, $v_o = 0$, $v_1 = ?$, $a = ?$

$v_2 = ?$, $t_3 - t_2 = ?$, $y_3 = 1.5$ m, $v_3 = 0$, $t_4 - t_2 = 4.0$ s, $v_4 = ?$

Target Quantity: a = acceleration of objects on planet

Relevant equations:

For constant acceleration

$$a_y = \frac{\Delta v_y}{\Delta t}$$

$$y_f = \frac{1}{2}a(\Delta t)^2 + v_{yo}\Delta t + y_o$$

Use equations for constant acceleration

$$a_y = \frac{\Delta v_y}{\Delta t} \qquad y_f = \frac{1}{2}a(\Delta t)^2 + v_{yo}\Delta t + y_o$$

During 3 different time intervals

t_o to t_1

$$a = \frac{v_1 - v_o}{t_1 - t_o} = \frac{v_1}{t_1}$$

$$y_1 = \frac{1}{2}a(t_1 - t_o)^2 + v_o(t_1 - t_o) + y_o = \frac{1}{2}at_1^2$$

t_2 to t_4

$$a = \frac{v_4 - (-v_2)}{t_4 - t_2} = \frac{v_4 + v_2}{t_4 - t_2}$$

$$y_4 = \frac{1}{2}a(t_4 - t_2)^2 + (-v_2)(t_4 - t_2) + y_o$$

$$y_4 = \frac{1}{2}a(t_4 - t_2)^2 - v_2(t_4 - t_2)$$

t_2 to t_3

$$a = \frac{v_3 - (-v_2)}{t_3 - t_2} = \frac{v_2}{t_3 - t_2}$$

$$-y_3 = \frac{1}{2}a(t_3 - t_2)^2 + (-v_2)(t_3 - t_2)$$

Step 3: Plan the solution

Step 3: Plan the solution

	unknowns
Find a From drop $y_1 = \frac{1}{2}a(t_1)^2$ [1]	a
Find y_1 from throw $y_1 = \frac{1}{2}a(t_4 - t_2)^2 - v_2(t_4 - t_2)$ [2]	y_1
Find v_2 from throw going up $-y_3 = \frac{1}{2}a(t_3 - t_2)^2 - v_2(t_3 - t_2)$ [3]	v_2
Find $t_3 - t_2$ from throw $a = \frac{v_3 - (-v_2)}{t_3 - t_2}$, $a = \frac{v_2}{t_3 - t_2}$ [4]	$t_3 - t_2$

4 unknowns, 4 equations — Can solve it

Execute the Plan:

Evaluate Solution:

Properly stated?

Unreasonable?

Complete?

Execute the Plan:

Solve [4] for $t_3 - t_2$

$$t_3 - t_2 = \frac{v_2}{a} \quad \text{Into [3]}$$

$$-y_3 = \frac{1}{2}a\left(\frac{v_2}{a}\right)^2 - v_2\left(\frac{v_2}{a}\right)$$

$$-y_3 = \frac{1}{2}\frac{v_2^2}{a} - \frac{v_2^2}{a}$$

$$y_3 = \frac{v_2^2}{2a}$$

$$\sqrt{2ay_3} = v_2 \quad \text{Into [2]}$$

$$y_1 = \frac{1}{2}a(t_4 - t_2)^2 - \sqrt{2ay_3}(t_4 - t_2) \quad \text{Into [1]}$$

$$\frac{1}{2}a(t_4 - t_2)^2 - \sqrt{2ay_3}(t_4 - t_2) = \frac{1}{2}a(t_1)^2$$

only unknown is a

Check units

$$\left[\frac{m}{\cancel{s^2}}\right][\cancel{s^2}] - \sqrt{\left[\frac{m}{\cancel{s^2}}\right][m]}[\cancel{s}] = \left[\frac{m}{\cancel{s^2}}\right][\cancel{s^2}]$$

$$[m] - [m] = [m] \quad \text{ok}$$

$$\frac{1}{2}a(t_4 - t_2)^2 - \sqrt{2ay_3}(t_4 - t_2) = \frac{1}{2}a(t_1)^2$$

$$\frac{1}{2}a(t_4 - t_2)^2 - \frac{1}{2}a(t_1)^2 = \sqrt{2ay_3}(t_4 - t_2)$$

$$\frac{1}{2}a\left[(t_4 - t_2)^2 - (t_1)^2\right] = \sqrt{2ay_3}(t_4 - t_2)$$

$$\frac{1}{4}a^2\left[(t_4 - t_2)^2 - (t_1)^2\right]^2 = 2ay_3(t_4 - t_2)^2$$

$$\frac{1}{4}a\left[(t_4 - t_2)^2 - (t_1)^2\right]^2 = 2y_3(t_4 - t_2)^2$$

$$a = 8y_3\frac{(t_4 - t_2)^2}{\left[(t_4 - t_2)^2 - (t_1)^2\right]^2}$$

check units

$$[a] = [m]\frac{1}{[s]^2} = \left[\frac{m}{s^2}\right] \quad \text{ok}$$

$$a = 8y_3\frac{(t_4 - t_2)^2}{\left[(t_4 - t_2)^2 - (t_1)^2\right]^2}$$

$$a = 8(1.5m)\frac{(4.0s)^2}{\left[(4.0s)^2 - (3.0s)^2\right]^2} = 3.9\frac{m}{s^2}$$

This is about 0.4g which is not Earth and is about right for Mars.

Evaluate Solution:

Properly stated?

Yes. Acceleration is in m/s²

Unreasonable?

No. It is a little less than half of the Earth's gravitational acceleration

Complete?

Yes. They are on Mars.

Replay

Approach: Constant acceleration kinematics

Drop object

Relationship between acceleration and height of cliff

Don't know height of cliff

Throw object

Relationship between acceleration and height of cliff

Don't know initial velocity

Know height above cliff

Relationship between initial velocity and height above cliff

Don't know time for max height

Relationship between time interval and velocity at max height

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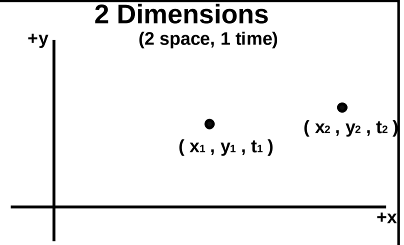
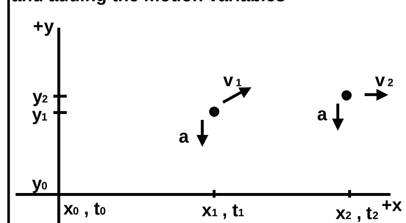
2 Dimensions

(2 space, 1 time)



Rearranging the position - time designations and adding the motion variables



Kinematics

Definition of average velocity $\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$

Definition of instantaneous velocity (velocity at a space - time point) $\vec{v} = \frac{d\vec{r}}{dt}$

Definition of average acceleration $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

Definition of instantaneous acceleration (acceleration at a space - time point) $\vec{a} = \frac{d\vec{v}}{dt}$

But now $\Delta \vec{r}$ and $\Delta \vec{v}$ are more complicated than in 1-dimension

Quantitative 2 -D Kinematics

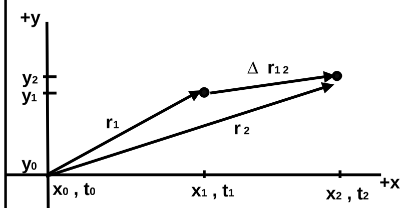
The Great Discovery

erpendicular parts of motion are independent

To solve a problem of an object moving

1. Break the motion into perpendicular parts
   Each 1 Dimension
2. Calculate everything you need
   (1 Dimensional Kinematics)
3. Put the perpendicular parts back together

Average velocity moving between 1 and 2 (qualitative)



$$\vec{\bar{v}}_{12} = \frac{\Delta \vec{r}_{12}}{\Delta t}$$

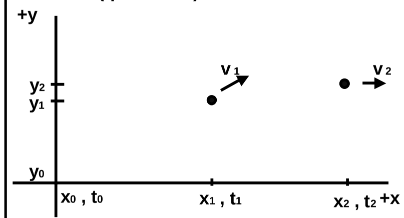
$$\Delta \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$\vec{\bar{v}}_{12}$ is in the direction of $\Delta \vec{r}_{12}$

$$\Delta \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r}_1 + \Delta \vec{r}_{12} = \vec{r}_2$$

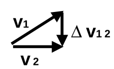
Average acceleration moving between 1 and 2 (qualitative)



$$\vec{\bar{a}}_{12} = \frac{\Delta \vec{v}_{12}}{\Delta t}$$

$$\Delta \vec{v}_{12} = \vec{v}_2 - \vec{v}_1$$

$\vec{\bar{a}}_{12}$ is in the direction of $\Delta \vec{v}_{12}$

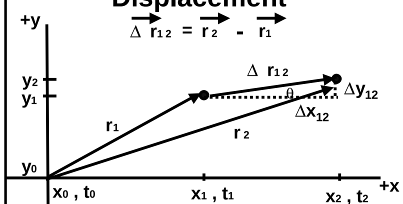


$$\Delta \vec{v}_{12} = \vec{v}_2 - \vec{v}_1$$

$$\vec{v}_1 + \Delta \vec{v}_{12} = \vec{v}_2$$

Displacement

$$\Delta \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$



Consider x and y displacements independently

$\vec{r}_1$ Is made of x_1 and y_1

$\vec{r}_2$ Is made of x_2 and y_2

x displacement : $\Delta x_{12} = x_2 - x_1$

y displacement : $\Delta y_{12} = y_2 - y_1$

Put the perpendicular parts together

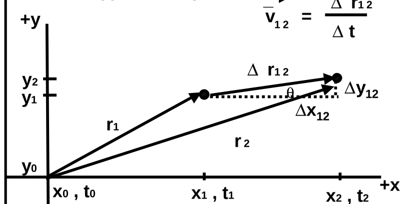
Displacement

size $\Delta r_{12}^2 = \Delta x_{12}^2 + \Delta y_{12}^2$

direction $\tan\theta = \frac{\Delta y_{12}}{\Delta x_{12}}$

Average velocity moving between 1 and 2 (qualitative)

$$\vec{\bar{v}}_{12} = \frac{\Delta \vec{r}_{12}}{\Delta t}$$



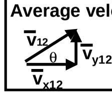
Consider perpendicular parts independently

$\bar{v}_{x12} = \frac{\Delta x_{12}}{\Delta t}$ $\Delta x_{12} = x_2 - x_1$

$\bar{v}_{y12} = \frac{\Delta y_{12}}{\Delta t}$ $\Delta y_{12} = y_2 - y_1$

Put the perpendicular parts together

Average velocity

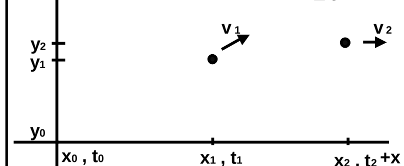


size $\bar{v}_{12}^2 = \bar{v}_{x12}^2 + \bar{v}_{y12}^2$

direction $\tan\theta = \frac{\bar{v}_{y12}}{\bar{v}_{x12}}$

Average acceleration moving between 1 and 2 (qualitative)

$$\vec{\bar{a}}_{12} = \frac{\Delta \vec{v}_{12}}{\Delta t}$$



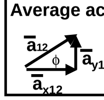
Consider perpendicular parts independently

$\bar{a}_{x12} = \frac{\Delta v_{x12}}{\Delta t}$ $\Delta v_{x12} = v_{x2} - v_{x1}$

$\bar{a}_{y12} = \frac{\Delta v_{y12}}{\Delta t}$ $\Delta v_{y12} = v_{y2} - v_{y1}$

Put the perpendicular parts together

Average acceleration



size $\bar{a}_{12}^2 = \bar{a}_{x12}^2 + \bar{a}_{y12}^2$

direction $\tan\phi = \frac{\bar{a}_{y12}}{\bar{a}_{x12}}$

Review

Difference between

Speed and Velocity

Difference between

displacement and distance

Displacement	Distance
Vector	Scalar
Know end points	Know path

Average Speed NOT = Average Velocity

Unless object moves in

Straight line in constant direction

Instantaneous Speed =

Magnitude of Instantaneous Velocity

Average Speed

$$\bar{s} = \frac{\text{distance}}{t_f - t_o} = \frac{\text{dist}}{\Delta t}$$

Instantaneous Speed

$$s = \lim_{\Delta t \to 0} \frac{\text{dist.}}{\Delta t}$$

Average Velocity

$$\overline{v_x} = \frac{x_f - x_o}{t_f - t_o} = \frac{\Delta x}{\Delta t}$$

Use instantaneous position to calculate average velocity

Instantaneous Velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

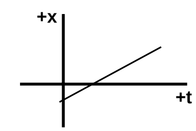
Special Cases

Constant Speed

$s = \bar{s}$

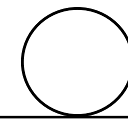
Constant Velocity

$v_x = \bar{v}_x$



If an object has a constant velocity
it always has a constant speed

If an object has a constant speed
it may not have a constant velocity



Average acceleration

$$\overline{a_x} = \frac{v_{xf} - v_{xo}}{t_f - t_o} = \frac{\Delta v_x}{\Delta t}$$

Use instantaneous velocity to calculate average acceleration

Instantaneous acceleration

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

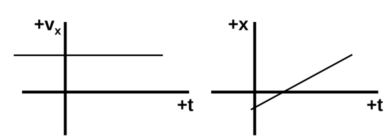
Special Cases:

Acceleration = 0

Constant velocity

average velocity = instantaneous velocity

$v_f = v_o$



Acceleration is Constant

average acceleration
= instantaneous acceleration

$$a_x = \frac{v_{x\ f} - v_{x\ o}}{t_f - t_o}$$

$$a_x = \frac{\Delta v_x}{\Delta t}$$

$$x_f = \frac{1}{2} a_x (\Delta t)^2 + v_{xo}\Delta t + x_o$$