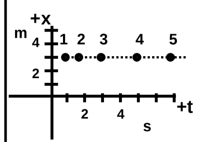
Velocity

Measure positions and times   Lab 1

Graph position vs time of an object



What was the motion of the object?

At each time it was measured, the object was in the same position

A theory   The object existed between the measured positions

What was the motion of the object?

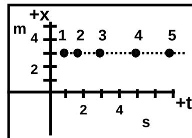
In words: The object did not move

In math: $x = 3$ m

What was the object's average velocity?

$v_{av} = \frac{(x_B - x_A)}{(t_B - t_A)}$ In words: The object's average velocity was zero

In math: $v_{av} = 0$



Do we need the theory (line) to determine average velocity?   No

What was the instantaneous velocity?

$v = \frac{dx}{dt}$ (slope of x vs t curve)

In words: The instantaneous velocity is zero

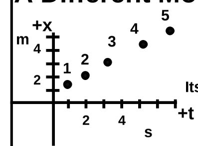
In math: $v = 0$

We need the theory (line) to determine instantaneous velocity

Average velocity comes directly from measurements

Instantaneous velocity comes from measurements and a theory

A Different Motion



What the motion of this object?

Its position changes with time.

Not good enough. How does the position change?

Does the average velocity help?

$v_{av} = \frac{x_f - x_0}{t_f - t_0}$ Which average velocity?

$v_{av} = \frac{x_5 - x_1}{t_5 - t_1}$ $v_{av} = \frac{x_4 - x_2}{t_4 - t_2}$ $v_{av} = \frac{x_3 - x_2}{t_3 - t_2}$

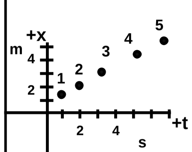
For this case, all average velocities are equal

Calculate the average velocity between position 2 and position 4

$v_{av} = \frac{x_4 - x_2}{t_4 - t_2}$ $v_{av} = \frac{4.5\text{m} - 2.0\text{m}}{5.2\text{s} - 2.0\text{s}} = 0.78$ m/s

Can you determine an average velocity at times other than those measured?

Motion

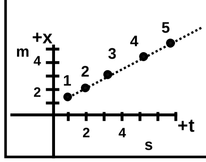


Does the average velocity describe the motion between measurements?

Depends on the actual motion

Was it this?

Or this?



The average velocity always describes the motion only if the velocity is constant

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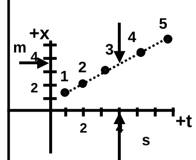
Instantaneous Velocity



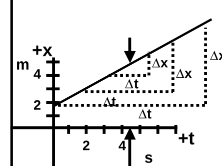
determine the instantaneous velocity at 4 seconds?

$v = \frac{dx}{dt}$ (slope of x vs t curve at 4 seconds)

he slope at a point on a curve is the slope of he tangent to the curve at that point

Finding the slope of a straight line at any point is easy because the slope of a straight line does not change.

Constant velocity



Determine the (instantaneous) velocity at 4.5 s

$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

$\frac{\Delta x}{\Delta t}$ everywhere the same for straight line

$\frac{dx}{dt} = \frac{\Delta x}{\Delta t}$ Only for a straight line

A constant average velocity for all time intervals means

The instantaneous velocity of an object equals its average velocity.

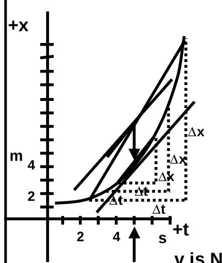
$v = \frac{\Delta x}{\Delta t}$ Only if v is constant

Usually this is NOT the case

Changing velocity

Usually the instantaneous velocity of an object is not constant

The position of the object changes differently for equal time intervals



$\frac{\Delta x}{\Delta t}$ Different at each time for a curve

Find the object's velocity at 5 s

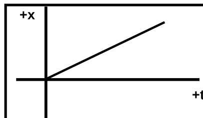
$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$\frac{dx}{dt}$ NOT $\frac{\Delta x}{\Delta t}$

v is NOT $\frac{\Delta x}{\Delta t}$ if v changes

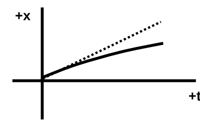
Instantaneous velocity at a given time is

The SLOPE of (the tangent to) position vs time curve at that instant of time



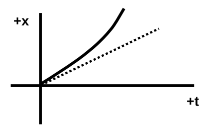
constant velocity

For equal time intervals an object has the same displacement



slowing down ?

For equal time intervals an object has a smaller and smaller displacement

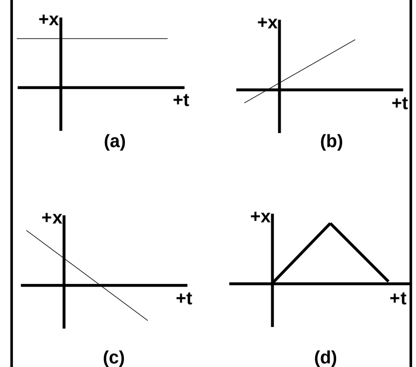


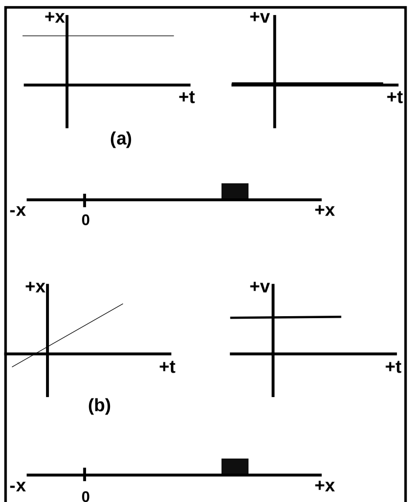
speeding up ?

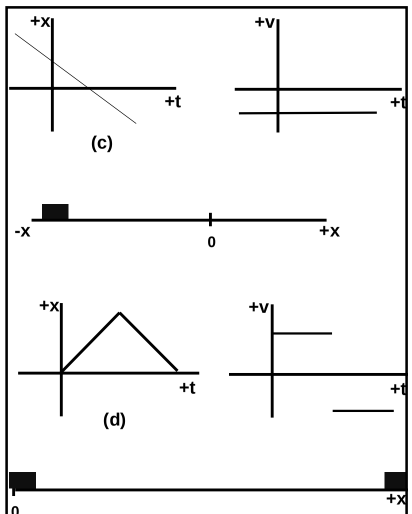
For equal time intervals an object has a greater and greater displacement

Velocity

If the position of an object changes in the graphs below, sketch a graph of the object's velocity vs. time.

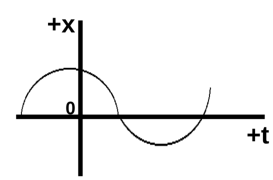






Motion

The graph below gives the motion of a car.



Indicate one place on the graph when the car is

(a) Changing direction?

(b) Speeding up?

(c) Slowing down?

(d) Travelling with constant velocity?

Describe this car's trip in words.

Acceleration:
speeding up and slowing down

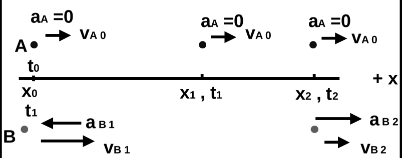
- Definition of average acceleration

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} \qquad \Delta \vec{v} \text{ is } \vec{v}_f - \vec{v}_i \qquad \Delta t \text{ is } t_f - t_i$$

- Definition of instantaneous acceleration (acceleration at a space - time point)

$$\vec{a} = \frac{d\vec{v}}{dt} \qquad \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Make a diagram of the motion



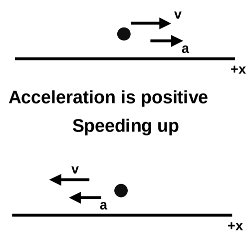
Describe what happened in words

Speeding Up
In one dimension

If the acceleration is in the same direction as the initial velocity

the acceleration has the same sign as the velocity

the object is speeding up.



Acceleration is positive

Speeding up

Acceleration is negative

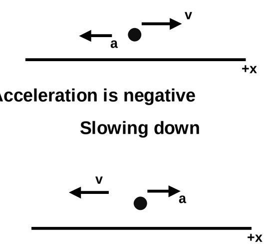
Speeding up

Slowing Down
In one dimension

If the acceleration is in the opposite direction as the initial velocity,

the acceleration has the opposite sign as the velocity

the object is slowing down



Acceleration is negative

Slowing down

Acceleration is positive

Slowing down

Acceleration in 1 Dimension

$$a_{av} = \frac{v_B - v_A}{(t_B - t_A)}$$

The sign of the acceleration gives the direction.

\+ means toward the + direction as you defined your coordinate system.

\- means toward the - direction as you defined your coordinate system.

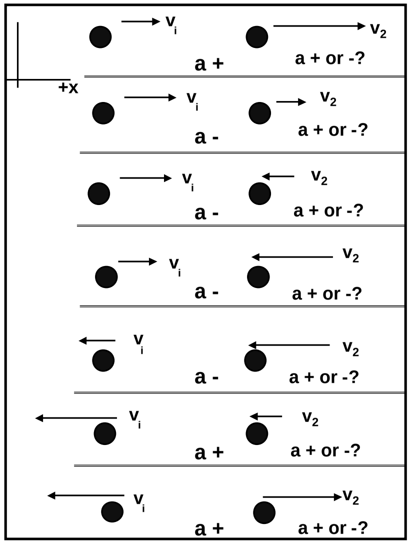
In one dimension (along the x axis):

If the velocity is +
the change of position is in the + x direction.

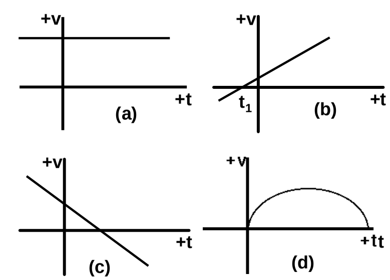
If the velocity is -
the change of position is in the - x direction.

If the acceleration is +
the change of velocity is along the + x direction

If the acceleration is -
the change in velocity is along the - x direction.



Acceleration



Describe the motion in each case.

(a) velocity +, constant
acceleration 0

(b) velocity starts -, slows down to 0 at t_1, becomes + at t_1, speeds up
acceleration +, constant

Calculations

Relate average quantities to instantaneous quantities

The diagram has only instantaneous quantities

Special Cases:

- Velocity is constant

  $\vec{v}_{av} = \vec{v}$ and $\vec{a}_{av} = \vec{a} = 0$

- Acceleration is constant (not 0)

  $\vec{a}_{av} = \vec{a}$ and $\vec{v}_{av}$ NOT $\vec{v}$

Mostly acceleration is not constant

$\vec{a}_{av}$ NOT $\vec{a}$ $\quad$ $\vec{v}_{av}$ NOT $\vec{v}$

The direction of acceleration is the same as the direction of the <u>difference</u> between

the final velocity and the initial velocity.

Acceleration is a vector

Apply Kinematics

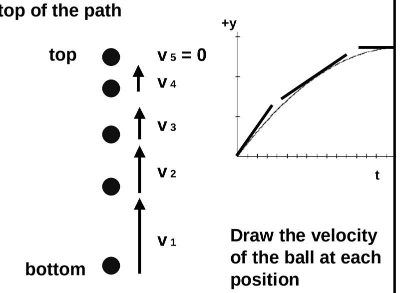
First application: Objects falling near the surface of the Earth

A ball is thrown straight up.

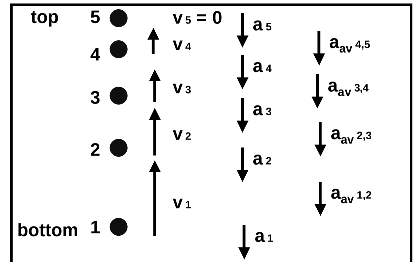
How does it move?

Constant velocity
Speeds up
Slows down

Draw a strobe picture of the motion of the ball from the moment it leaves the hand until the top of the path



Draw the velocity of the ball at each position



Draw the acceleration of the ball at each position

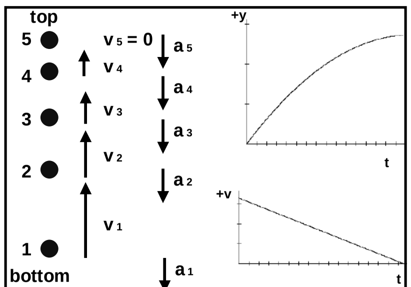
Draw the average acceleration between each position (picture).

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

To draw the instantaneous acceleration at each position

Imagine taking the time interval between positions to 0

$$\vec{a} = \frac{d\vec{v}}{dt}$$

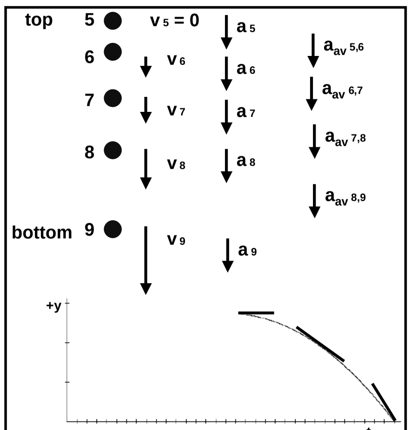


The velocity of the object decreases in magnitude until it reaches the top.

While the object moves up, the acceleration is down.

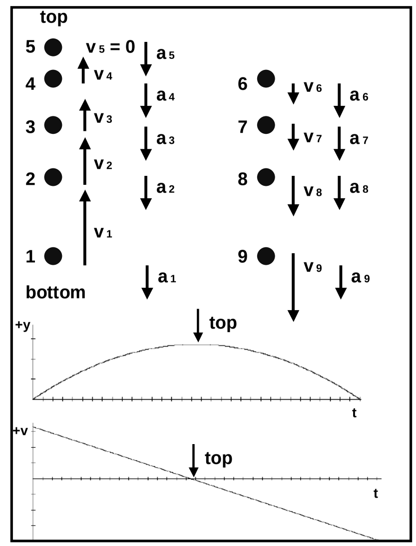
At the top, the velocity is zero and the acceleration is NOT. It is down.

Now lets do the same thing to describe the motion of the object moving from the top of its path back to the bottom



The velocity of the object increases in magnitude as it moves from top to bottom

While the object moves down, the acceleration is down.



Free Fall

Velocity changes direction and magnitude

Acceleration is always down

Even when moving up, the object is “falling”

Free fall

Measurements show that the acceleration is constant for objects in free fall near the surface of the Earth

Is the magnitude of the acceleration Different for different objects?

Need observations

Drop different objects and measure their acceleration

Easy test

Drop (initial velocity 0) two objects from the same height

If accel. is the same, they hit at same time

Gravitational Acceleration

Careful measurements show

ALL objects in free fall near the surface of the Earth have the same acceleration

we give this acceleration a special name g

The magnitude of g is about 10 m / s²

The direction of g is down

g is the gravitational acceleration of objects near the surface of the Earth.

g is NOT "gravity" !!

g is does not change even it the distance of the object from the Earth changes as long as it is near the surface of the Earth.

Constant Acceleration

A very useful special case

Mathematically describe the motion of an object with constant acceleration

equation of motion

$a = \frac{dv}{dt}$ Always true

$v = \frac{dy}{dt}$ Always true

Combining the definitions for v and a

$a = \frac{d}{dt}(\frac{dy}{dt})$ Always true

If a is constant

Differentiating y with respect to t twice

Gives a constant (does not depend on t)

What mathematical function describes y ?

Solving the Equation of Motion

The best technique

An educated guess

Want a function y to satisfy $a = \frac{d}{dt}(\frac{dy}{dt})$

What function of t can you take 2 derivatives giving a function that does not depend on t

$y = at^2$ Gives a constant but not a 2 a

How about ?

$y = \frac{1}{2}at^2$

How about ?

$y = \frac{1}{2}at^2 + bt$ b is a constant

How about ?

$y = \frac{1}{2}at^2 + bt + c$ c is a constant

Anything else possible ?

What are the Constants?

a is the acceleration, what are b and c ?

Use units to help

$y = \frac{1}{2}at^2 + bt + c$

y is the position of the object [m]

a is the acceleration of the object [m/s²]

t is the time the object is at that position [s]

[m] = a [s²] + b [s] + c

c is in [m] (position, displacement, distance)

But what one?

b is in $\frac{m}{s}$ (velocity, speed)

But what one?

Check some special cases where you know the answer

Special Cases

When t = 0, what is the position of the object?

The initial position

When t = 0, $y = y_0$

The equation for the position of the object

$y = \frac{1}{2}at^2 + bt + y_0$

When t = 0, y =c

Therefore $c = y_o$

Now we need b

We might also know the initial velocity

When t = 0, $v = v_0$

Get the velocity by taking the time derivative of position $\frac{dy}{dt}$

To find b differentiate the equation for position as a function of time $\frac{dy}{dt}$

$$y = \frac{1}{2}at^2 + bt + y_0$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{1}{2}at^2 + bt + y_0\right)$$

$$\frac{dy}{dt} = at + b$$

$$v = at + b$$

Suppose you know the initial velocity

When $t = 0$, $v = v_0$

When $t = 0$, $v_0 = b$

$y = \frac{1}{2}at^2 + v_0 t + y_0$ When $t_o = 0$

Check to see if this is correct by doing two derivatives to get acceleration

$\frac{dy}{dt} = at + v_o$ $\frac{d}{dt}(at + v_o) = a$ Ok!

Review

If $t_0 \neq 0$,

$$y = \frac{1}{2}a(\Delta t)^2 + v_0\Delta t + y_0$$

Equation of motion of object with constant acceleration

Comes from the definitions of

velocity

acceleration

using a little calculus

$$a = \frac{d}{dt}\left(\frac{dy}{dt}\right)$$

Undoing two time derivatives gives

t^2

two constants

v_0

y_0