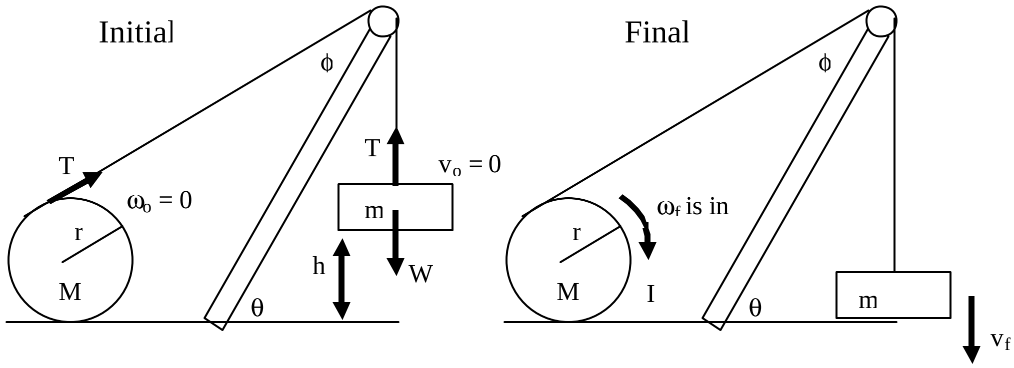
Solutions for Quiz 4

Problem 2



$\theta = 60^o$
$\phi = 40^o$
Know
m
h
v_f
$v_o = 0$
M
r
$\omega_o = 0$

Find the moment of inertial of the drum as a fraction of the moment of inertia of a ring (Mr^2.) $\boxed{I = f Mr^2}$

Use conservation of energy: $E_f - E_i = E_{in} - E_{out}$

System: container, drum, cable, Earth

Initial time: container at maximum height. Assume it is not moving. All energy of the system is gravitational potential energy.

$$E_i = mgh$$

Final time: just before container hits the ground. Both the drum and the container have kinetic energy.

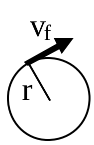
$$E_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

No significant external forces on the system so $E_{transfer} = 0$.

$$\boxed{\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 - mgh = 0}$$

Assume the moment of inertial of the pulley is small (massless pulley approximation).
Assume the frictional force on the pulley and on the drum is negligible.
Assume the cable mass can be neglected.

The outside of the drum moves a distance that is equal to the distance the cable moves down in the same time. The speed of the outside of the drum is the same as the speed of the cable.



$$\boxed{\omega_f = \frac{v_f}{r}}$$

Target: f

Plan		unknowns
Find f		f
$I = fMr^2$	[1]	I
Find I		
$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 - mgh = 0$	[2]	ω_f
Find ω_f		
$\omega_f = \frac{v_f}{r}$	[3]	

3 unknowns, 3 equations – ok to solve

Solve [3] for ω_f and put into [2]

$$\omega_f = \frac{v_f}{r}$$

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I\left(\frac{v_f}{r}\right)^2 - mgh = 0 \quad \text{solve for I and put into [1]}$$

$$I = \frac{2mgh - mv_f^2}{\left(\frac{v_f}{r}\right)^2}$$

$$\frac{2mgh - mv_f^2}{\left(\frac{v_f}{r}\right)^2} = fMr^2$$

$$\boxed{\frac{2mgh - mv_f^2}{M(v_f)^2} = f}$$

Evaluate:

Check units: $\frac{[J]-[J]}{[J]} = [1] = [f]$ This is correct since a fraction has no units.

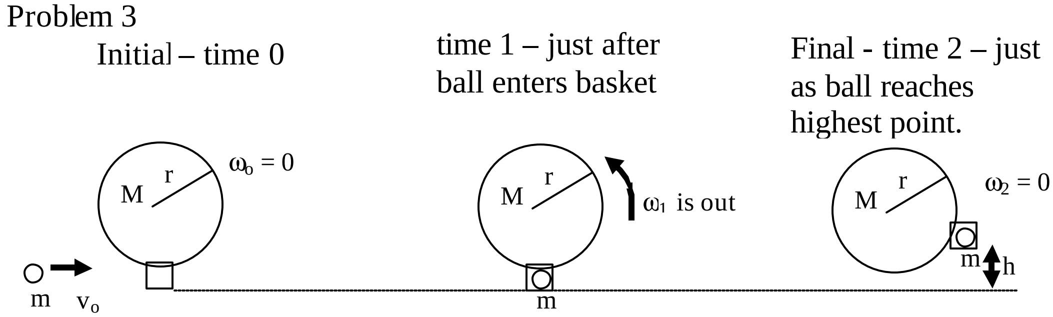
The moment of inertia of the drum is larger if f is larger.

If the mass of the container (m) is larger, f must be larger for the container to hit the ground with the same velocity. This is reasonable since the container initially has more energy and so a larger amount of energy would have be rotational energy of the drum.

If the maximum speed that the container could hit the ground was greater, f would be smaller. This is reasonable because less energy would need to be in the rotational kinetic energy of the drum. The moment of inertia of the drum could be smaller.

If the mass of the drum (M) is larger, f could be smaller for the container to hit the ground with the same velocity. This is reasonable since a larger drum mass gives a larger rotational energy of the drum.

Problem 3



Know:
r, M, m, v_o , $I=(1/2)Mr^2$

Find maximum height of the ball (h).

Use conservation of angular momentum between the initial time and time 1.

$\vec{L}_1 - \vec{L}_o = \Delta\vec{L}_{transfer}$

System: ball and disk

Initial time: angular momentum of the ball. $\vec{L}_o = \vec{r} \times \vec{p}$.

$L_o = mv_o$ direction is out. Take out to be +.

Time 1: angular momentum of the ball and disk. $\vec{L}_1 = I_{ball}\vec{\omega}_1 + I_{disk}\vec{\omega}_1$.

$$I_{ball} = mr^2, \ I_{disk} = \frac{1}{2}Mr^2$$

$\Delta\vec{L}_{transfer} = 0$

$$\boxed{mr^2\omega_1 + \frac{1}{2}Mr^2\omega_1 - mv_o = 0}$$

Use conservation of energy between the time 1 and final time: $E_f - E_i = E_{in} - E_{out}$

System: ball, disk, Earth

Time 1: ball is in the basket. Both the ball and the disk have kinetic energy.

$$E_1 = \frac{1}{2}I_{ball}\omega_1^2 + \frac{1}{2}I_{disk}\omega_1^2$$

Final time: ball reaches highest point. All energy of the system energy is gravitational potential energy.

$$E_f = mgh$$

No significant external forces on the system so $E_{transfer} = 0$.

$$\boxed{\frac{1}{2}\left(mr^2\right)\omega_1^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega_1^2 - mgh = 0}$$

Assume the mass of the basket is small (massless basket).

Assume the frictional force on the disk is negligible.

Target: h

Plan	unknowns
Find h	h
$\frac{1}{2}\left(mr^2\right)\omega_1^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega_1^2 - mgh = 0$ [1]	ω_1

Find ω_1

$$mr^2\omega_1 + \frac{1}{2}Mr^2\omega_1 - rmv_o = 0 \qquad [2]$$

2 unknowns, 2 equations – ok to solve

Solve [2] for ω_1 and put into [1]

$$mr^2\omega_1 + \frac{1}{2}Mr^2\omega_1 - rmv_o = 0$$

$$\omega_1 = \frac{rmv_o}{mr^2 + \frac{1}{2}Mr^2}$$

$$\omega_1 = \frac{mv_o}{mr + \frac{1}{2}Mr}$$

$$\left(\frac{1}{2}m + \frac{1}{4}M\right)r^2\omega_1^2 - mgh = 0$$

$$\left(\frac{1}{2}m + \frac{1}{4}M\right)r^2\left(\frac{mv_o}{mr + \frac{1}{2}Mr}\right)^2 - mgh = 0$$

$$\frac{1}{2}\left(m + \frac{1}{2}M\right)\left(\frac{mv_o}{m + \frac{1}{2}M}\right)^2 - mgh = 0$$

$$\frac{1}{2}\frac{mv_o^2}{m + \frac{1}{2}M} = gh$$

$$\boxed{\frac{1}{2g}\frac{mv_o^2}{m + \frac{1}{2}M} = h}$$

Evaluate:

Check units: $\dfrac{1}{\left[\dfrac{m}{s^2}\right]}\dfrac{[kg]\left[\dfrac{m}{s}\right]^2}{[kg]+[kg]}=\dfrac{1}{\left[\dfrac{m}{s^2}\right]}\left[\dfrac{m}{s}\right]^2=[m]=[h]$ This is correct since meters is a units of height.

If the speed of the ball (v_o) increases, h increases. This is reasonable since the system starts with more energy so the final energy would also be larger even if a significant amount of the initial energy goes into changing the system's internal energy in this perfectly inelastic collision.

If the mass of the disk (M) increases, h decreases. This is reasonable since if the disk were infinitely massive, it would not go anywhere after the collision.

Answers to Conceptual Questions

1. b
2. e
3. b
4. d
5. d
6. a
7. d
8. a
9. a
10. c