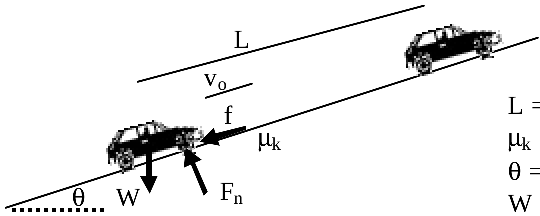
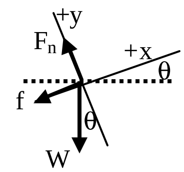
2.





L =100 ft
μ_k = 0.60
$\theta = 10^o$
W = 2050 lb

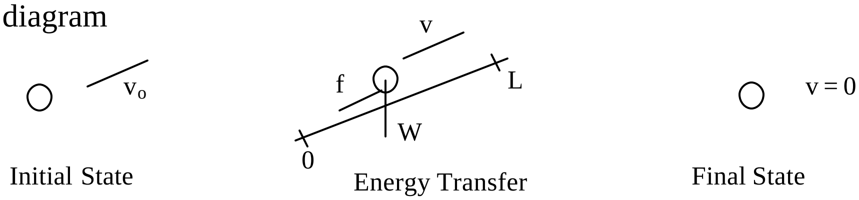
$W_x = W\sin\theta$
$W_y = W\cos\theta$
$f = \mu F_n$

Is the initial velocity of the car greater than 30 mph?
Approach:
Use conservation of energy.

System: car
Initial time: before you apply brakes.
Final time: after you stop.
Energy output from car by frictional force and component of gravitational force (weight) along direction of motion. Neglect the air resistance.

Use dynamics to get frictional force.

Energy diagram



$$E_i = \frac{1}{2}mv_o^2 \qquad\qquad E_f = 0$$

$$E_{output} = \int_0^L W_x dx + \int_0^L f dx = W\sin\theta L + fL$$

Conservation of Energy: E_f - E_i = E_{input} - E_{output}

.

$$0 - \left(\frac{1}{2}mv_o^2\right) = -\left(W\sin\theta L + fL\right) \text{ and } W=mg$$

$$\left(\frac{1}{2}\frac{W}{g}v_o^2\right) = \left(W\sin\theta L + fL\right)$$

Get the frictional force from the normal force. Use dynamics to get the normal force.

$$\sum F_y = \left(F_n - W\cos\theta\right) = 0$$

Plan unknowns

Find v_o v_o

$$\left(\frac{1}{2}\frac{W}{g}v_o^2\right) = \left(W\sin\theta L + fL\right) \quad [1] \qquad f$$

Find f

$f = \mu F_n$ [2] F_n

Find F_n

$(F_n - W\cos\theta) = 0$ [3]

3 unknown, 3 equations

$$(F_n - W\cos\theta) = 0$$

$$F_n = W\cos\theta \text{ into [2]}$$

$$f = \mu_k W\cos\theta \text{ into [1]}$$

$$\left(\frac{1}{2}\frac{W}{g}v_o^2\right) = \left(W\sin\theta L + \mu_k W\cos\theta L\right)$$

$$v_o^2 = 2g(\sin\theta L + \mu_k \cos\theta L)$$

$$v_o = \sqrt{2gL(\sin\theta + \mu_k\cos\theta)}$$

check units

$$v_o = \sqrt{\left[\frac{m}{s^2}\right][m]} = \left[\frac{m}{s}\right] \text{ correct for velocity}$$

$$\boxed{v_o = \sqrt{2\left(32\frac{ft}{s^2}\right)(100ft)\left(\sin 10^o + 0.6\cos 10^o\right)} = 70 \text{ ft/s}}$$

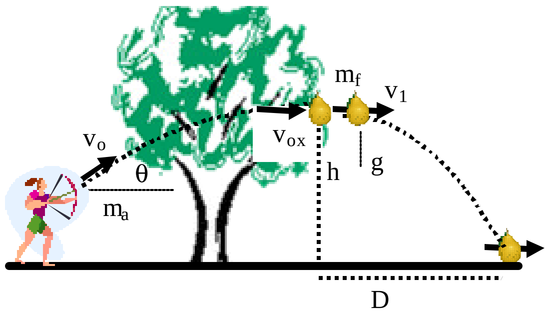
$70\frac{ft}{s}\left(\frac{1mi}{5280ft}\right)\left(\frac{60s}{1min}\right)\left(\frac{60min}{1hr}\right) = 48mph$ This is greater than the 30 mph speed limit. You were speeding.

The velocity has the correct units of ft/s.

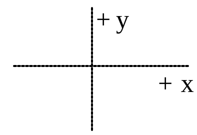
If θ were 90^o, the car would be falling straight down. The answer becomes $v_o = \sqrt{2gL}$ which does not depend on the coefficient of friction. This is reasonable if the car is in free fall.

If the skid length increases, the initial velocity would be greater. This is reasonable since it would take a longer distance to stop the car if it were going at a higher speed.

3.



Find the distance the fruit hits the ground as a function of the initial speed and angle of the arrow, the height of the fruit on the tree, the mass of the fruit and the mass of the arrow.



Approach:
Use kinematics to calculate D from v_1 and h. After the arrow enters the fruit and it flies through the air with a constant vertical acceleration and constant horizontal velocity.

Since the arrow sticks in the fruit, there is a large internal energy change in the arrow-fruit system.

Use conservation of momentum to relate the arrow's velocity just before it hits the fruit to the velocity of the fruit just after the arrow enters it.

Since the arrow hits the fruit at the highest part of its path, its velocity is horizontal and equal to the horizontal component of the arrow's initial momentum. The horizontal component of the velocity of the arrow is constant. $v_{ox} = v_o \cos\theta$

Conservation of momentum $p_{fx} - p_{ix} = p_{input\ x} - p_{output\ x}$

System: arrow + fruit

Initial time: just before arrow enters fruit $p_{ix} = m_a v_{ox} = m_a v_o \cos\theta$

Final time: just after arrow enters fruit $p_{fx} = (m_a + m_f)v_1$

No momentum transfer in x direction since no forces in x direction.

$$(m_a + m_f)v_1 - m_a v_o \cos\theta = 0$$

Flight of arrow and fruit after leaving the tree

Constant horizontal velocity:

average x component of velocity = instantaneous x component of velocity.

$$v_1 = v_{av} = \frac{\Delta x}{\Delta t} = \frac{D}{\Delta t}$$

Constant vertical acceleration:

average y component of acceleration = instantaneous y component of acceleration.

$$-g = v_{av} = \frac{\Delta v_x}{\Delta t}$$

and $y_f = \frac{1}{2}a_y(\Delta t)^2 + v_{oy}(\Delta t) + y_o$

$$0 = -\frac{1}{2}g(\Delta t)^2 + h$$

Plan unknowns

Find D

$$v_1 = \frac{D}{\Delta t} \qquad [1] \qquad \Delta t,\ v_1$$

Find Δt

$$0 = -\frac{1}{2} g(\Delta t)^2 + h \qquad [2]$$

Find v_1

$$(m_a + m_f)v_1 - m_a v_o \cos\theta = 0 \qquad [3]$$

3 unknowns, 3 equations

$$(m_a + m_f)v_1 - m_a v_o \cos\theta = 0$$

$$v_1 = \frac{m_a v_o \cos\theta}{(m_a + m_f)} \text{ into [1]}$$

$$0 = -\frac{1}{2} g(\Delta t)^2 + h$$

$$\Delta t = \sqrt{\frac{2h}{g}} \text{ into [1]}$$

$$\frac{m_a v_o \cos\theta}{(m_a + m_f)} = \frac{D}{\sqrt{\frac{2h}{g}}}$$

$$\boxed{\frac{m_a v_o \cos\theta}{(m_a + m_f)}\sqrt{\frac{2h}{g}} = D}$$

Evaluate:

Check units:

$$\frac{[kg]\left[\frac{m}{s}\right]}{[kg]}\sqrt{\frac{[m]}{\left[\frac{m}{s^2}\right]}} = \left[\frac{m}{s}\right]\sqrt{\left[s^2\right]} = [m] \quad \text{correct for a distance}$$

If the initial speed (v_o) of the arrow is greater, the fruit goes a greater distance (D). This is reasonable.

If the angle the arrow is shot is 90^o, the distance the fruit goes is zero. That is reasonable because if the arrow is shot straight up, the fruit must be directly over the hero to hit it. The fruit will then fall straight down after it is hit.

If the mass of the fruit is larger, then it does not go as far from the tree. That is reasonable.

The higher the fruit is off the ground, the farther it falls from the tree. That is reasonable since its horizontal component of velocity is the same but it has a longer time to travel.

Conceptual Questions:

1. b
2. a
3. a
4. a
5. c
6. e
7. d
8. c
9. b
10. b