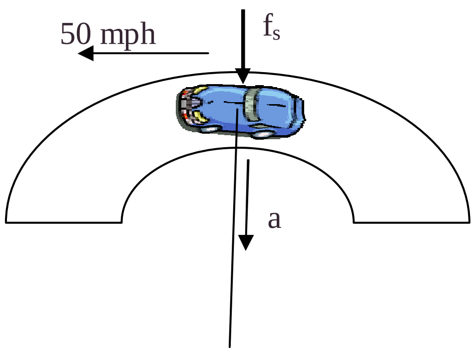
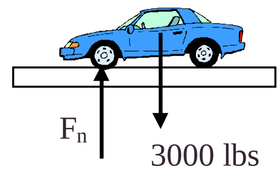
Solution: Quiz Individual Problems

Problem 2.

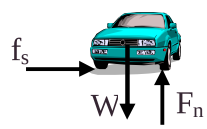
Top view



Side view



Front view



fs is the static frictional force on the car (tires) from the road.

a is the acceleration of the car that is moving at a constant speed (v) of 50 mph

W is the weight of the car = 3000lbs

What is the smallest coefficient of static friction that will give the frictional force necessary for the car to travel in a circle with a radius of 0.05 mi?

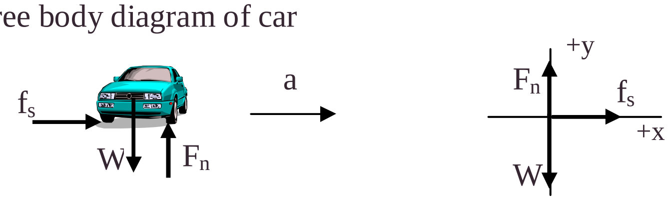
Approach: Use Newton's 2nd Law to relate the forces on the car to its acceleration and mass.

The car travels with uniform circular motion so its acceleration is v^2/r.

Can get the maximum static frictional force from the normal force. That is the largest frictional force for a given coefficient of static friction. Using the maximum static frictional force will give the minimum coefficient of static friction.

Choose a coordinate system with one axis along the radius of the circle (along the acceleration). One axis will be vertical.

Free body diagram of car



W = 3000 lbs
V = 50 mi/hr
r = 0.05 mi
g = 9.8 m/s^2

$$\sum F_x = f_s = ma$$

$$\sum F_y = F_n - W = 0$$

$$f_s = \mu_s F_n$$

$$a = \frac{v^2}{r}$$

$$W = mg$$

Target: μ_s

Plan | unknowns

Find μ_s		μ_s
$f_s = \mu_s F_n$	1	f_s , F_n
Find f_s		
$f_s = m\frac{v^2}{r}$	2	m
Find m		
$W = mg$	3	
Find F_n		
$F_n - W = 0$	4	

4 unknowns, 4 equations. Ok to solve.

Execute:

[4] $F_n - W = 0$

$F_n = W$ put into 1

[1] $f_s = \mu_s W$

[3] $W = mg$

$\frac{W}{g} = m$ put into 2

[2] $f_s = \frac{W}{g}\frac{v^2}{r}$ put into 1

[1] $\frac{W}{g}\frac{v^2}{r} = \mu_s W$

$$\boxed{\frac{v^2}{rg} = \mu_s}$$

Check units $\frac{\left[\frac{m}{s}\right]^2}{[m]\left[\frac{m}{s^2}\right]} = [1]$ correct since μ_s has no units

$$\frac{\left(50\frac{mi}{hr}\right)^2}{(0.05mi)\left(9.8\frac{m}{s^2}\right)} = \mu_s = 5102\left(\frac{(mi)(s^2)}{(m)(hr^2)}\right)\left(\frac{hr}{60\,min}\right)^2\left(\frac{min}{60s}\right)^2\left(\frac{1000m}{\frac{5}{8}mi}\right) = 0.63$$

$\mu_s = 0.63$

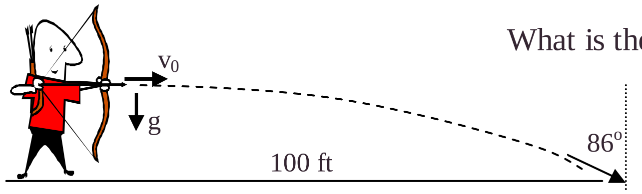
Evaluate:

If the speed gets larger, it takes a larger coefficient of static friction for the car to get around the corner. This is reasonable since it is more difficult to go around a corner if you go faster.

If the radius of the turn is smaller, it takes a larger coefficient of static friction for the car to get around the corner. This is reasonable since a corner with a smaller radius is a tighter turn.

0.63 is a typical value for coefficient of static friction for surfaces that do not slide easily.

Problem 3



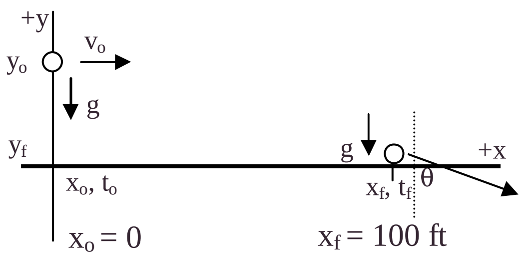
What is the initial velocity of the arrow?

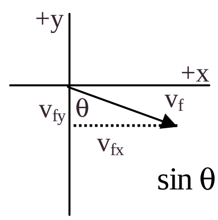
Approach: Perpendicular components of motion are independent.

Horizontal motion: constant velocity

Vertical motion: constant acceleration

Use the definition of velocity and acceleration.





$x_o = 0$	$x_f = 100$ ft
$t_o = 0$	$t_f = ?$
$y_o = ?$	$y_f = 0$
$v_o = ?$	$v_f = ?$
$g = 32$ ft/s^2	$\theta = 86^o$

$$\sin\theta = \frac{v_{fx}}{v_f} = \frac{v_{ox}}{v_f}$$

$$\cos\theta = \frac{v_{fy}}{v_f}$$

Target: v_0

Horizontal: $v_x = v_{xav} = \dfrac{x_f - x_0}{t_f - t_0} = \dfrac{x_f}{t_f}$

$v_x = v_{fx} = v_{ox}$

Vertical: $a = a_{av} = -g = \dfrac{-v_{fy} - v_{0y}}{t_f - t_0} = \dfrac{-v_{fy}}{t_f}$

$$y_f = -\frac{1}{2}g(t_f - t_0)^2 + v_{0y}(t_f - t_0) + y_0 = -\frac{1}{2}g(t_f)^2 + y_0 = 0$$

Plan		unknowns
Find v_o		v_o
$\sin\theta = \dfrac{v_{ox}}{v_f}$	1	v_f
Find v_f		
$\cos\theta = \dfrac{v_{fy}}{v_f}$	2	v_{fy}
Find v_{fy}		
$-g = \dfrac{-v_{fy}}{t_f}$	3	t_f
Find t_f		
$v_{ox} = \dfrac{x_f}{t_f}$	4	

4 unknowns, 4 equations. Ok to solve.

Execute:

[4] $v_{ox} = \dfrac{x_f}{t_f}$

$$t_f = \frac{x_f}{v_{ox}}$$ put into 3

[3] $g = \dfrac{v_{fy}}{\dfrac{x_f}{v_{ox}}}$

$$g\frac{x_f}{v_{ox}} = v_{fy}$$ put into 2

[2] $\cos\theta = \dfrac{g\dfrac{x_f}{v_{ox}}}{v_f}$

$$v_f = \frac{g\dfrac{x_f}{v_{ox}}}{\cos\theta}$$ put into 1

[1] $\sin\theta = \dfrac{v_{ox}}{\dfrac{g\dfrac{x_f}{v_{ox}}}{\cos\theta}}$

$$\sin\theta \frac{g\dfrac{x_f}{v_{ox}}}{\cos\theta} = v_{ox}$$

$$\sin\theta g\frac{x_f}{v_{ox}} = v_{ox}\cos\theta$$

$$\sin\theta g\frac{x_f}{\cos\theta} = v^2_{ox}$$

$$\boxed{\sqrt{gx_f \tan\theta} = v_{ox}}$$

Check units $\sqrt{\left[\dfrac{m}{s^2}\right][m]} = \sqrt{\left[\dfrac{m^2}{s^2}\right]} = \left[\dfrac{m}{s}\right]$ correct units for a velocity

$$\sqrt{\left[32\frac{ft}{s^2}\right][100ft]\tan 86^\circ} = v_{ox} = 214 \text{ ft/s}$$

Evaluate:

If the distance the arrow goes gets larger, the initial speed of the arrow must increase. This is reasonable.

If the gravitational acceleration were smaller, the distance would be larger for the same initial speed of the arrow. This is reasonable since the arrow would take a longer time to fall to the ground.

A fast runner can go 100 m in 10 sec or about 30 ft/s. The arrow's initial speed is about 7 times faster than a person can run. Reasonable for an arrow. It is about 5 times slower than the speed of a bullet, about the speed of sound or 1000ft/s. Also reasonable for an arrow.

Conceptual Questions:

1. d
2. d
3. a
4. a
5. b
6. e
7. e
8. b
9. d
10. a