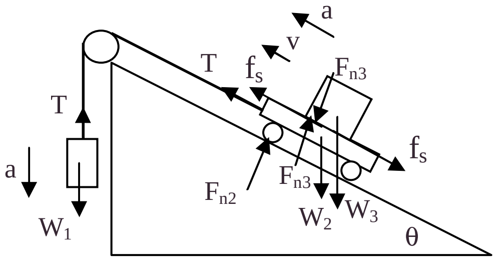
Solution: Quiz 2 Group Problem



The hanging weight is object 1 (mass m_1)
The cart is object 2 (mass m_2)
The luggage container is object 3 (mass m_3)

What is the mass of the hanging weight as a function of the mass of the cart, the luggage container, the angle of the ramp, and the coefficient of static friction so that the luggage container and the cart have the same acceleration?

Approach: Use Newton's 2nd Law to relate the forces on each object to its acceleration and mass. All 3 objects have the same acceleration if the container does not fall off the cart.

From Newton's 3rd Law the frictional force of the cart on the container equals the frictional force of the container on the cart. Also the normal force of the cart on the container equals the normal force of the container on the cart.

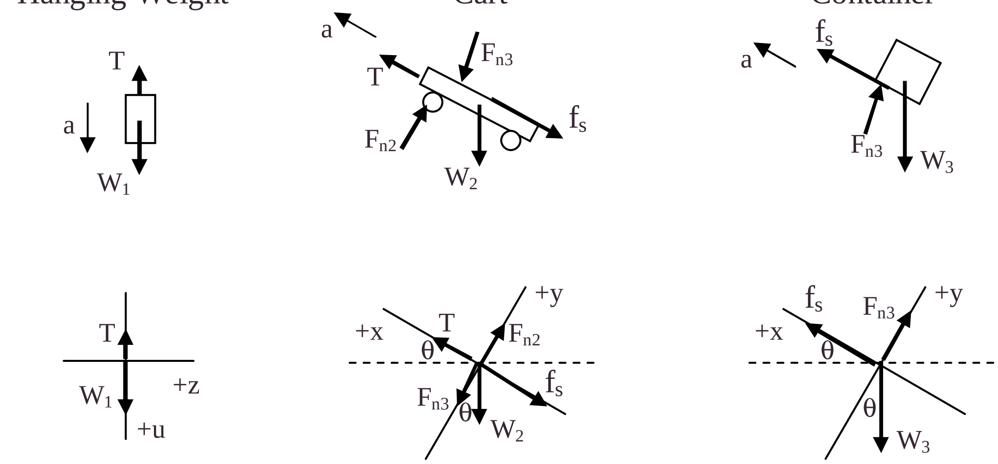
Since we want the container to move with the cart, it must have the same acceleration as the cart. Since the surface of the container does not move relative to the surface of the cart, the frictional force is static friction.

Assume a massless cable and massless, frictionless pulley. The cable exerts the same force at both ends.

Choose a coordinate system with one axis along the ramp.

Free body diagrams

Hanging Weight | Cart | Container



Hanging Weight

$$\sum F_u = m_1 g - T = m_1 a$$

Cart

$$\sum F_x = T - f_s - m_2 g \sin\theta = m_2 a$$

$$\sum F_y = F_{n2} - F_{n3} - m_2 g \cos\theta = 0$$

Container

$$\sum F_x = f_s - m_3 g \sin\theta = m_3 a$$

$$\sum F_y = F_{n3} - m_3 g \cos\theta = 0$$

$f_s = \mu_s F_{n3}$ for maximum static frictional force

Target: m_1

Plan		unknowns
Find m_1		m_1
$m_1 g - T = m_1 a$	1	T, a
Find T		
$T - \mu_s F_{n3} - m_2 g \sin\theta = m_2 a$	2	F_{n3}
Find F_{n3}		
$F_{n3} - m_3 g \cos\theta = 0$	3	
Find a		
$\mu_s F_{n3} - m_3 g \sin\theta = m_3 a$	4	

4 unknowns, 4 equations. Ok to solve.

Execute:

[4] $\mu_s F_{n3} - m_3 g \sin\theta = m_3 a$

$$\frac{\mu_s F_{n3} - m_3 g \sin\theta}{m_3} = a$$ put into 1 and 2

[1] $$m_1 g - T = m_1 \frac{\mu_s F_{n3} - m_3 g \sin\theta}{m_3}$$

[2] $$T - \mu_s F_{n3} - m_2 g \sin\theta = m_2 \frac{\mu_s F_{n3} - m_3 g \sin\theta}{m_3}$$

[3] $F_{n3} - m_3 g \cos\theta = 0$

$F_{n3} = m_3 g \cos\theta$ put into 1 and 2

[1] $$m_1 g - T = m_1 \frac{\mu_s m_3 g \cos\theta - m_3 g \sin\theta}{m_3} = m_1 g(\mu_s \cos\theta - \sin\theta)$$

[2] $$T - \mu_s m_3 g \cos\theta - m_2 g \sin\theta = m_2 \frac{\mu_s m_3 g \cos\theta - m_3 g \sin\theta}{m_3} = m_2 g(\mu_s \cos\theta - \sin\theta)$$

$T = \mu_s m_3 g \cos\theta + m_2 g \sin\theta + m_2 g(\mu_s \cos\theta - \sin\theta)$ put into 1

$$m_1 g - \mu_s m_3 g \cos\theta - m_2 g \sin\theta - m_2 g(\mu_s \cos\theta - \sin\theta) = m_1 g(\mu_s \cos\theta - \sin\theta)$$

$$m_1 g - m_1 g(\mu_s \cos\theta - \sin\theta) = \mu_s m_3 g \cos\theta + m_2 g \sin\theta + m_2 g(\mu_s \cos\theta - \sin\theta)$$

$$m_1 = \frac{\mu_s m_3 \cos\theta + m_2 \sin\theta + m_2(\mu_s \cos\theta - \sin\theta)}{1 - (\mu_s \cos\theta - \sin\theta)}$$

$$m_1 = \frac{\mu_s m_3 \cos\theta + m_2 \mu_s \cos\theta}{1 - (\mu_s \cos\theta - \sin\theta)}$$

$$\boxed{m_1 = (m_3 + m_2)\frac{\mu_s \cos\theta}{1 - (\mu_s \cos\theta - \sin\theta)}}$$

Evaluate:

Units are ok since the sum of two masses has units of mass and everything else in the equation has no units.

m_1 depends on the sum of m_2 and m_3. This is not unreasonable since if either the mass of the cart or the mass of the container increased, the mass of the hanging weight would have to increase it you wanted to keep the same acceleration.

If the angle of the ramp with the horizontal were 90 degrees, m_1 would be 0. This is not unreasonable since at 90 degrees, the cart and the container would be going straight down. They would move together if they were in free fall (both with the acceleration g). Thus the cable should not pull on them at all. This requires no hanging weight ($m_1 = 0$).