Already accomplished

- Read Text Chapter 1
- Read Text Chapter 2
- Read Text Chapter 3
- Read CPS (Competent Problem Solver) Chap. 3
- Worked Text Problems Chap. 1- 3
- Worked CPS Problems Chap. 3
- Problems from Laboratory I & 2

Next 2 Weeks

Connect Motion (Chapter 2 & 3) with Interactions (Chapter 4 & 5) using Vectors

- Read Text Chapter 4 & 5
- Read CPS Chapter 4
- Work problems in Chapter 4
- Work problems in Chapter 5
- Work problems in CPS Chapter 4
- Problems from Laboratory 3 & 2

notes from quiz 1

- Both problems on individual quiz were from homework
  - Problem 87 – Chapter 2
  - Problem 40 – Chapter 3
- Common Difficulties leading to low scores
  - Organization
    - » Undefined symbols
    - » Equations from "nowhere"
    - » No explanations about why an equation would apply
    - » Poor pictures (no physics content)
    - » Poor or non-existent diagrams
    - » Solving for the wrong quantity
    - » Solution logic not explained
    - » No algebraic plan to get a solution
    - » Difficulty following algebraic plan to get an answer
  - Physics
    - » No distinction between average velocity and instantaneous velocity.
    - » Use of geometry and trig instead of physics
    - » A lot of time wasted thinking about and doing irrelevant things.
    - » No comparison of answer to other experience or logic with explanation

What to Do

Don't worry - Act

- Neither Physics nor good problem solving is natural.
  - Learn how and understand why.
  - Solving problems in a professional manner will feel uncomfortable for a while.
    - » Like learning to drive.
  - Solving problems without clear written communication for others is useless.
    - » OK for a hobby – Not for a profession.
- Improvement comes through practice and repetition.
  - Always practice solving a problem as if it were a test.
    - » At home
    - » In discussion section.
  - Always start from basic principles (the equations we give you.)
    - » Understand them completely
      - What the symbols really mean
      - Under what conditions they are true
      - Under what conditions they are useful.
  - Always solve problems by writing a logical and complete solution from the beginning.
    - » Never use "scratch paper".

What to Do- Discussion Section

- Get a perfect score on the next quiz group problem (and a higher score on the individual part too).
  - In discussion section practice writing a perfect paper.
    - » Make sure every member of your group comes prepared.
      - You need everyone to contribute!
    - » Make sure every member of the group agrees on every step.
      - If every member of your group does not completely understand how to solve the problem from what is written on the paper
        - It is either wrong or poorly explained. (both give a poor quiz grade)
    - » Practice writing as little as possible that explains as much as needed.
      - USE pictures and diagrams ("A picture is worth 1000 words.")
    - » Practice your group roles. To get a perfect score you need
      - A manager to keep track of time and progress.
      - A recorder checker to write an organized solution and make sure that everyone agrees with each step.
      - A skeptic to make sure that the solution does not assume that someone can "read your mind".
      - Everyone guiding the solution.

What to Do- At Home

- Solve all suggested problems as if it were a test.
  - Read the text first.
  - Write down the smallest number of fundamental principles in the chapter using equations where possible.
    - » Often one equation per chapter.
    - » Almost never more than three equations.
    - » The equations used in the lecture and on example problem solutions.
    - » Ask if in doubt.
      - Me
      - TAs
  - Attempt to solve at least one problem every day
    - » Only use equations representing the fundamental principles.
    - » Always write the solution in a logical and well explained manner from the beginning.
    - » Do not solve one way and try to "write it up".
    - » Work fast (no more than 20 minutes/problem)
    - » Do not refer back to the text.
    - » Do not look at the answer or solution outline.
    - » If stuck or unsure, get help. (email for direct questions & fast response)
      - Re-read the chapter & lecture notes.
      - Study group
      - TAs
      - Me
    - » If you got help, work another problem of the same difficulty in the same section.

What to Do- In Lab

- Come prepared
  - Read textbook
  - Pass pre-quiz
  - Read assigned laboratory problems
    - » Write down things unclear from the reading
  - Read relevant appendices
  - Write up methods questions for assigned problems.
  - Write up the prediction as if you were solving a test problem.
- Compare your prediction solution and methods question answers with your group.
  - Discuss and resolve differences and things you were unsure of.
- Make sure that the qualitative behavior of objects in the lab agrees with your "common sense"
  - Exploration section is very important
  - If there is a disagreement, recognize it and resolve it by discussion with other group members and TA.
- Make sure that the quantitative behavior of objects agrees with your predictions.
  - Understand the functions you use to fit the data.
    - » Relationship to fundamental principles
    - » Meaning of all coefficients.
  - Finish all analysis and make conclusion before doing next problem.
- Practice working rapidly and efficiently as a group.

What to Do- In Lecture

- Understand the need for the fundamental principles presented
  - Determine the connection to other things you know.
  - Check your understanding of them by answering
    - How do you know it is true?
    - Under what conditions does it apply?
    - What is it useful for?
- Stay at least one step ahead of example problem solutions
  - How is what is presented the same as what you would do?
  - How is what is presented different from what you would do?
    - How important are the differences?
- Do not read my lecture notes in class.
  - Go over them before class to review previous lectures
    - Write down any questions you have and ask them
  - Go over them after class to reinforce important issues or techniques for you.

What to Do- In Office Hours

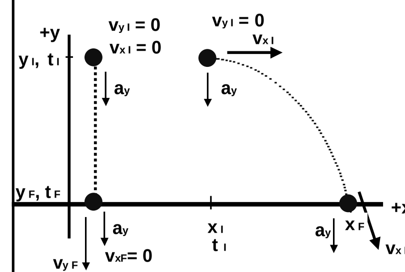
- Come for help when stuck or unsure
  - Leave time for at least ½ hour per problem.
  - Come at times that are unpopular if you can
  - Try different TAs and find one that is most in tune with what you need.
  - Come with a small study group if you can
    - Discuss the help you were given among yourselves afterwards.
- Show how you solved the problem from the beginning
  - Do not expect anyone to be able to jump into the middle of your solution
- Do not allow someone to show you how they would solve the problem.
  - Insist they see how you were trying to solve it.
  - Insist they use your logic or show you why your logic is wrong.

Combining Horizontal Motion and Vertical Motion

How does vertical motion depend on horizontal motion ?

Suppose you throw an object horizontally and at the same time drop an identical object.

Which object hits the ground first?



Conclusions

Time to hit the ground is ***same*** whether you throw it horizontally or you drop it.

The launch conditions for the vertical (y) motion are the same

- vertical initial positions (y_I) same
- vertical initial velocities (v_{yI}) same
- vertical accelerations same ($a_y = g$).

The object moves vertically from y_I to y_F the same way no matter what happens horizontally. Thus it takes the same time.

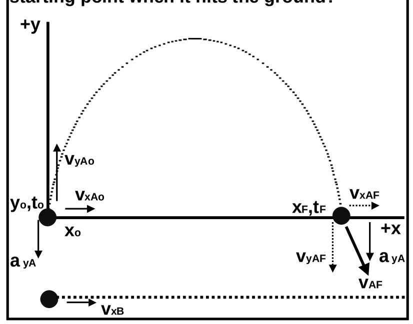
The motion of objects in the vertical direction is independent of their motion in the horizontal direction

Is the motion of objects in the horizontal direction independent of their motion in the vertical direction?

Does Horizontal Motion Depend on Vertical Motion?

Two objects move with the same constant horizontal velocity but one object has a vertical velocity and a vertical acceleration.

Which object ends up farthest from the starting point when it hits the ground?



Conclusion

Horizontal displacement of an object is the ***same*** if it has the same horizontal velocity whether or not it has a vertical velocity.

The launch conditions for the horizontal (x) motion are the same

- horizontal initial positions (x_I) are same
- horizontal initial velocities (v_{xI}) are same
- horizontal accelerations are same ($a_x = 0$).

The object moves horizontally from x_I to x_F the same way no matter what happens vertically. Thus it takes the same time.

The motion of objects in the horizontal direction is independent of their motion in the vertical direction

Projectile Motion: the Theory

Horizontal motion : v_x is constant

- Does ***not*** depend on properties (size, mass, density, ...) of the object.
- Does ***not*** depend on the vertical motion
- Depends ***only*** on initial horizontal velocity

Vertical motion: a_y is constant (called g)

- Does ***not*** depend on properties (size, mass, density, ...) of the object.

Does ***not*** depend on the horizontal motion

- Depends ***only*** on initial vertical velocity and vertical acceleration.

Projectile motion is ***completely*** determined by the launch conditions (v_{xI} and v_{yI}).

The motion of a baseball, a bullet, and a ballet dancer differ ***only*** because the launch velocities differ.

The above statements are clearly false if air resistance is important

What Causes this Motion?

What interacts with a thrown object?
(remember we assumed the interaction with the air is negligible)

The only interaction is with the Earth

Called Gravity

Direction of Gravitational Interaction of a thrown object with the Earth

Vertical - Down

How do we know?

Vertical motion: constant acceleration

Horizontal motion: zero acceleration

Guess a theory:

An interaction of one object with another causes that object to accelerate in the direction of the interaction.

Force

A more precise way describing an interaction

WHAT WE KNOW ABOUT FORCES

An object cannot change its velocity unless a force is exerted on it by another object

A force on an object is ALWAYS caused by another object.

Indicators of a Force on an object

- Contact with another object
- The object is accelerating
- The object is not accelerating but known forces are unbalanced

Is It A Force?

If you think a force acts on an object you MUST

- Identify another object causing the force.
- Identify the direction of the force.
- Identify the type of the force

Contact
Gravitational
Tension
Spring
Frictional
Magnetic
Electric

Force A is the GRAVITATIONAL PULL of the EARTH on the BALL.

Theory of Forces

The sum of the x components of Forces

ΣF_x

affects ONLY the x component of acceleration

ΣF_x causes a_x

The sum of the y components of Forces

ΣF_y

affects ONLY the y component of acceleration

ΣF_y causes a_y

The sum of the z components of Forces

ΣF_z

affects ONLY the z component of acceleration

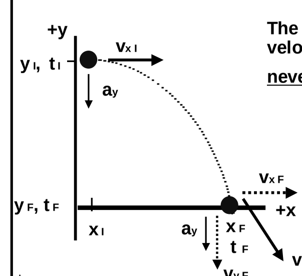
ΣF_z causes a_z

Mathematical Description

Need a coordinate system so mathematics makes sense.

Call x the horizontal coordinate and y the vertical coordinate.

The positive direction is defined on the drawing of the coordinate system



The horizontal velocity ***never*** changes

$\Delta y = y_F - y_I$

$\Delta t = t_F - t_I$

$\Delta x = x_F - x_I$

Horizontal Motion

Horizontal motion: v_x is constant

$a_x = 0$

Because there is no interaction with the object in the x direction

use the definition of average velocity

$$v_{x\,av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}$$

$$v_{x\,av} = v_x$$

Vertical Motion

Vertical motion: a_y is constant

v_y is changing

Because the object interacts with the Earth in the x direction

use the definition of acceleration

$$a_y = \frac{dv_y}{dt}$$

$$a_y = \frac{d}{dt}\left(\frac{dy}{dt}\right)$$

$$y_f = \frac{1}{2}a(\Delta t)^2 + v_{y0}\Delta t + y_0$$

use the definition of average acceleration

$$a_{y\,av} = \frac{\Delta v_y}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$$

$$a_y = a_{y\,ave}$$

Relevant Equations

No horizontal interaction

$v_x = \frac{\Delta x}{\Delta t}$

Horizontal motion

v_x constant

A vertical interaction

Vertical motion

$y = \frac{1}{2}a(\Delta t)^2 + v_{y0}\Delta t + y_0$ — a_y constant

$a_y = \frac{\Delta v_y}{\Delta t}$ — v_y changes

Δt is the same for horizontal and vertical motion

Just one object

Vectors

Mathematics to deal with independent perpendicular parts is called vectors

Perpendicular parts are called components.

Vector quantities

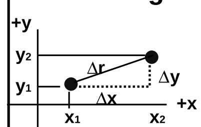
Position: x, y; $\vec{r} = x\hat{i} + y\hat{j}$	**Displacement:** $\Delta x, \Delta y$; $\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$
Velocity: v_x, v_y; $\vec{v} = v_x\hat{i} + v_y\hat{j}$	**Velocity change:** $\Delta v_x, \Delta v_y$; $\Delta\vec{v} = \Delta v_x\hat{i} + \Delta v_y\hat{j}$
Acceleration: a_x, a_y; $\vec{a} = a_x\hat{i} + a_y\hat{j}$	

Time does ***not*** have components.

The x motion occurs during the same time interval as the y motion.

Time is called a scalar

Taking Motion Apart



The object moves from position 1 to position 2

The magnitude of the displacement is Δr

The x position changes from x_1 to x_2.
The x displacement is Δx

The y position changes from y_1 to y_2.
The y displacement is Δy

Displacement Δr, is the "sum" of Δx and Δy

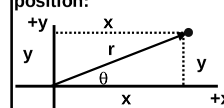
Δr NOT= $\Delta x + \Delta y$

From the drawing, Δx, Δy, and Δr make up a right triangle. That is one reason we use perpendicular coordinates.

Pythagorean, $(\Delta r)^2 = (\Delta x)^2 + (\Delta y)^2$

Components

position:



$r^2 = x^2 + y^2$

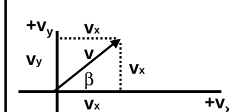
$\sin\theta = \frac{y}{r}$

$y = r\sin\theta$

$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

Velocity:



$v^2 = v_x^2 + v_y^2$

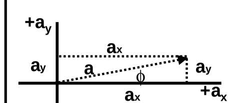
$\sin\beta = \frac{v_y}{v}$

$v_y = v\sin\beta$

$\cos\beta = \frac{v_x}{v}$

$v_x = v\cos\beta$

Acceleration:



$a^2 = a_x^2 + a_y^2$

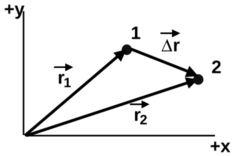
$\sin\phi = \frac{a_y}{a}$

$a_y = a\sin\phi$

$\cos\phi = \frac{a_x}{a}$

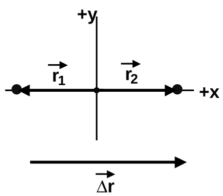
$a_x = a\cos\phi$

Change of Position



$\vec{\Delta r} = \vec{r_2} - \vec{r_1}$

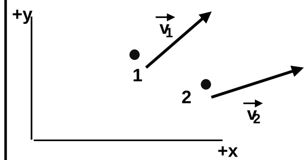
What must be added to r_1 to give r_2 ?

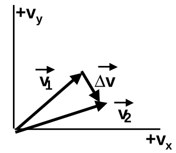


$r_1 = r_2$

What is $\vec{\Delta r}$

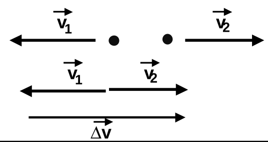
Change of Velocity





$\vec{\Delta v} = \vec{v_2} - \vec{v_1}$

What must be added to v_1 to give v_2 ?



$v_1 = v_2$

What is $\vec{\Delta v}$

Motion and Interactions

Type of motion results from the interaction of other objects with the object of interest

For simplicity assume only one interaction

No interaction
- Constant velocity
  - 1-D or straight line motion

Constant interaction along velocity
- Changing velocity
  - 1-D or straight line motion

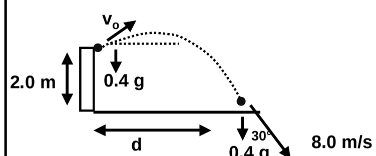
Constant interaction not along velocity
- Changing velocity
  - 2-D projectile motion

Changing interaction in direction but not magnitude - always perpendicular to the velocity

Example

You are serving on a citizen's panel to evaluate a proposal to search for life on Mars. A team of biologists has suggested that Martian life might be very fragile and decompose quickly in the heat from the Mars lander. They suggest that any search for life should begin at least 9 meters from the base of the lander. This biology team has designed a probe that is shot from the lander by a spring mechanism in the lander 2.0 meters above the surface of Mars. To return the data, the probe cannot be more than 11 meters from the bottom of the lander. Additional data acquisition and biological considerations require the probe to impact the surface with a velocity of 8.0-m/s at an angle of 30 degrees from the vertical. Can this probe work as designed? The Martian gravitational acceleration is 0.40g.

Focus



Question:

Is the distance the probe hits between 9 and 11 m?

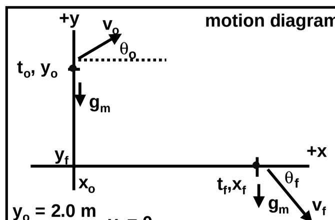
Approach:

Use kinematics.

Horizontal motion independent of vertical motion.

Horizontal: constant velocity
- No force on the object in the horizontal direction
- Ignore air resistance

Vertical: constant acceleration
- Only the gravitational force down

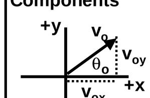


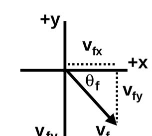
$y_o = 2.0$ m
$x_o = 0$
$t_o = 0$
$v_o = ?$

$y_f = 0$
$x_f = ?$
t_f ?
$g_m = 0.40g$

$v_f = 8.0$ m/s
$\theta_f = 60°$

Components





$\sin\theta_o = \frac{v_{oy}}{v_o}$

$\cos\theta_o = \frac{v_{ox}}{v_o}$

$\sin\theta_f = \frac{v_{fy}}{v_f}$

$\cos\theta_f = \frac{v_{fx}}{v_f} = \frac{v_{ox}}{v_f}$

Target: x_f

Horizontal : constant velocity

$v_x = \frac{x_f - x_o}{t_f - t_o} = \frac{x_f}{t_f}$ $\quad v_{xo} = v_{xf} = v_x$

Vertical: constant acceleration

$$-g_m = \frac{-v_{fy} - v_{oy}}{t_f}$$

$$0 = -\frac{1}{2} g_m t_f^2 + v_{oy} t_f + y_o$$

Plan

		unknowns
Find x_f		x_f
$v_x = \frac{x_f}{t_f}$	[1]	v_x, t_f
Find v_x		
$\cos\theta_f = \frac{v_x}{v_f}$	[2]	
Find t_f		
$0 = -\frac{1}{2} g_m t_f^2 + v_{oy} t_f + y_o$	[3]	v_{oy}
Find v_{oy}		
$-g_m = \frac{-v_{fy} - v_{oy}}{t_f}$	[4]	v_{fy}
Find v_{fy}		
$\sin\theta_f = \frac{v_{fy}}{v_f}$	[5]	no new unknowns

5 unknowns, 5 equations ok

Execute the Plan

[5] $v_f \sin\theta_f = v_{fy}$ Into [4]

$$-g_m = \frac{-v_f \sin\theta_f - v_{oy}}{t_f}$$

$v_{oy} = -v_f \sin\theta_f + g_m t_f$ Into [3]

$$0 = -\frac{1}{2} g_m t_f^2 + (-v_f \sin\theta_f + g_m t_f) t_f + y_o$$

$$0 = \frac{1}{2} g_m t_f^2 - v_f t_f \sin\theta_f + y_o$$

instead of solving this for t_f and put into

$$v_f \cos\theta_f = \frac{x_f}{t_f}$$

reverse the steps

$$t_f = \frac{x_f}{v_f \cos\theta_f}$$

$$0 = \frac{1}{2} g_m \left(\frac{x_f}{v_f \cos\theta_f}\right)^2 - v_f \frac{x_f}{v_f \cos\theta_f} \sin\theta_f + y_o$$

$$0 = \frac{1}{2} g_m \left(\frac{x_f}{v_f \cos\theta_f}\right)^2 - v_f \frac{x_f}{v_f \cos\theta_f} \sin\theta_f + y_o$$

$$0 = \frac{g_m}{2 v_f^2 \cos^2\theta_f} x_f^2 - x_f \tan\theta_f + y_o$$

check units

$$\frac{\left[\frac{m}{\cancel{s^2}}\right]}{\left[\frac{\cancel{m^2}}{\cancel{s^2}}\right]}\left[\cancel{m^2}\right] + [m] + [m] = [m]$$

ok

Use quadratic equation to solve for x_f

$$x_f = \frac{\tan\theta_f \pm \sqrt{\tan^2\theta_f - 4\left(\frac{g_m}{2 v_f^2 \cos^2\theta_f}\right) y_o}}{2\left(\frac{g_m}{2 v_f^2 \cos^2\theta_f}\right)}$$

$$x_f = \frac{\tan\theta_f \pm \sqrt{\tan^2\theta_f - 2\left(\frac{g_m}{v_f^2 \cos^2\theta_f}\right) y_o}}{\left(\frac{g_m}{v_f^2 \cos^2\theta_f}\right)}$$

put in numbers

$$x_f = \frac{\tan(60^\circ) \pm \sqrt{\tan^2(60^\circ) - 2\left(\frac{\left(0.40 \times 9.8\frac{m}{s^2}\right)}{\left(8.0\frac{m}{s}\right)^2 \cos^2(60^\circ)}\right)(2.0m)}}{\left(\frac{\left(0.40 \times 9.8\frac{m}{s^2}\right)}{\left(8.0\frac{m}{s}\right)^2 \cos^2(60^\circ)}\right)}$$

$$x_f = \frac{0.58 \pm 0.34}{0.25\left(\frac{1}{m}\right)}$$

$x_f = 3.76m$ or $0.96m$

This will not work, back to the drawing board

Evaluate

The distance is given in the correct units, m.

The distance is not unreasonable since something on the order of meters was predicted by the design.

From the answer:

$$x_f = \frac{\tan\theta_f \pm \sqrt{\tan^2\theta_f - 2\left(\frac{g_m}{v_f^2 \cos^2\theta_f}\right) y_o}}{\left(\frac{g_m}{v_f^2 \cos^2\theta_f}\right)}$$

The distance is infinite if g_m is zero

Reasonable since that means there is no gravitational force to pull the probe down.

The distance is zero if v_f is zero

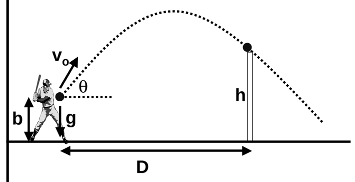
Reasonable since that means there is no horizontal component of velocity initially.

The question is answered, the plan will not work

Example

You are writing software for a new computer baseball game. You must determine an equation with which the computer can determine if a batter hits a home run based on the size of the ballpark and the way the ball is hit. The computer chooses reasonable random numbers for the distance of the bat from the ground when it hit the ball, the angle from the horizontal that the ball leaves the bat, and the initial speed of the ball off the bat. The height of the outfield wall and its distance from the batter is also known.

Focus the problem



Question: Is the vertical position of the ball greater than the height of the wall when it is at the horizontal position of the wall?

Approach:

vertical and horizontal motion are independent

No force in the horizontal direction

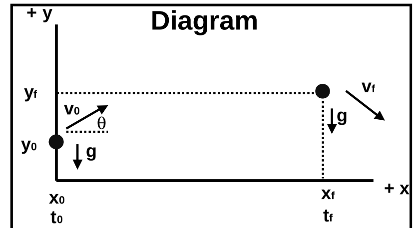
Neglect air resistance

Horizontal component of velocity is constant

Gravitational force in the vertical direction

Acceleration is vertical and constant

Diagram



$y_0 = b$ θ = known $x_f = D$

$t_0 = 0$ $a_y = g$ $y_f = ?$

$x_0 = 0$ $t_f = ?$

v_0 = known $v_f = ?$

Target quantity is y_f Is y_f > height of wall?

Quantitative relationships:

horiz: const. veloc.   vert: const. accel.

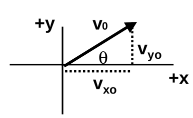
$$v_x = \frac{x_1}{t_1}$$

$$y = \frac{1}{2}(-g)(t_1)^2 + v_{y0}(t_1) + y_0$$

$$-g = \frac{v_{y1} - v_{y0}}{t_1}$$

Components:

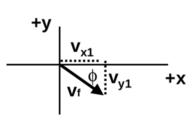
At initial time



$\sin\theta = \frac{v_{yo}}{v_0}$

$\cos\theta = \frac{v_{xo}}{v_0}$

At final time



$\sin\phi = \frac{v_{y1}}{v_f}$

$\cos\phi = \frac{v_{xf}}{v_1} = \frac{v_{xo}}{v_1}$

Plan

unknowns

Find y_f y_f

vertical motion of ball

$$y_1 = \frac{1}{2}(-g)(t_1)^2 + v_{y0}(t_1) + y_0 \quad \boxed{1}$$ t_1, v_{y0}

Find v_{0y}

$$\sin\theta = \frac{v_{y0}}{v_0} \quad \boxed{2}$$

Find t_f

horizontal motion of ball

$$v_x = \frac{x_1}{t_1} \quad \boxed{3}$$ v_x

Find v_x

$$\cos\theta = \frac{v_x}{v_0} \quad \boxed{4}$$ no new unknowns

4 unknowns, 4 equations ok

Execute the Plan

$\boxed{4}$ $v_0 \cos\theta = v_x$ Into [3]

$$v_o \cos\theta = \frac{x_1}{t_f}$$

$$t_1 = \frac{x_1}{v_o \cos\theta}$$ Into [1] along with [2]

$$y_1 = \frac{1}{2}(-g)\left(\frac{x_1}{v_o\cos\theta}\right)^2 + v_0 \sin\theta\left(\frac{x_1}{v_o \cos\theta}\right) + y_0$$

$$y_1 = -\frac{1}{2}g\left(\frac{x_1}{v_o\cos\theta}\right)^2 + \sin\theta\left(\frac{x_1}{\cos\theta}\right) + y_0$$

check units

$$\left[\frac{m}{s^2}\right]\frac{[m]^2}{\left[\frac{m}{s}\right]^2} + [m] + [m] = [m]$$ OK

Recap

A

constant vertical acceleration

$$y_f = \frac{1}{2}a(t_f - t_0)^2 + v_{y0}(t_f - t_0) + y_0$$

B

definition of sin θ for initial velocity vector

$$\sin\theta = \frac{v_{yo}}{v_0}$$

C

definition of average horizontal velocity
constant horizontal velocity

$$v_x = \frac{x_f - x_0}{t_f - t_0}$$

$$v_x = v_{xo}$$

D

definition of cos θ for initial velocity vector

$$\cos\theta = \frac{v_x}{v_0}$$

Evaluate

$$y_1 = -\frac{1}{2}g\left(\frac{D}{v_o\cos\theta}\right)^2 + \sin\theta\left(\frac{D}{\cos\theta}\right) + y_0$$

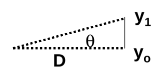
Is this unreasonable?

Imagine some easy situations where you know the answer

If g = 0 (no gravitational force)

$$y_1 = \sin\theta\left(\frac{D}{\cos\theta}\right) + y_0$$

$$y_1 - y_o = D\tan\theta$$



Straight line motion correct if no gravitational force pulling down

Reasonable

$$y_1 = -\frac{1}{2}g\left(\frac{D}{v_o\cos\theta}\right)^2 + \sin\theta\left(\frac{D}{\cos\theta}\right) + y_0$$

If angle = 0

$$y_1 = -\frac{1}{2}g\left(\frac{D}{v_o}\right)^2 + y_0$$

$v_o = v_x$

So $D/v_o = t_1$

$$y_1 - y_o = -\frac{1}{2}g(t_1)^2$$

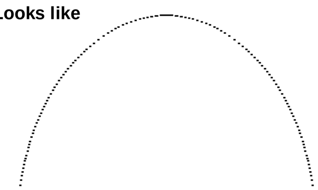
Correct for falling straight down

Reasonable for no initial vertical component of velocity

Projectile Motion Trajectory

Determine the path (trajectory) of a projectile.

Looks like



The path (trajectory) described by an equation gives the relationship between

The object's horizontal position
and
The object's vertical position

What determines the trajectory?

- the initial velocity of the object
- the acceleration of the object
- the initial position of the object

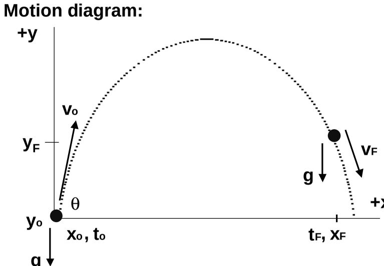
Want an equation that has
Vertical position on one side
and
horizontal position, g, v_o, and θ
on the other side

Approach:

- Use definition of velocity
- Use definition of acceleration
- Horizontal motion independent of vertical motion
  - Horizontal motion: const. velocity
  - Vertical motion: const. acceleration

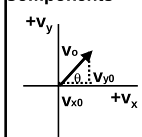
Approximation: ignore interaction with air

Motion diagram:

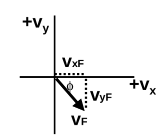


$x_o = 0$	v_o = known	x_F = known
$y_o = 0$	θ = known	y_F = ?
$t_o = 0$	g = known	t_F = ?
		v_F = ?

Components



initial velocity



final velocity

Target quantity: y_F as function of x_F, v_o, θ, g

Quantitative relationships (Tools):

horizontal	vertical
Constant velocity	Constant acceleration
v_x is constant	a_y is constant
$v_x = \frac{\Delta x}{\Delta t}$	$a_y = \frac{\Delta v_y}{\Delta t}$
	$y_F = \frac{1}{2}a_y(\Delta t)^2 + v_{yo}\Delta t + y_0$

Velocity components:

Initial time	Final time
$\sin\theta = \frac{v_{y0}}{v_o}$	$\sin\phi = \frac{v_{yF}}{v_F}$
$\cos\theta = \frac{v_{x0}}{v_o}$	$\cos\phi = \frac{v_{xF}}{v_F} = \frac{v_{xo}}{v_F}$

Plan

unknowns

Find y_F

object's vertical motion

$$y_F = -\frac{1}{2}gt_F^2 + v_{yo}t_F \quad \boxed{1} \qquad v_{y0} \quad t_F$$

Find v_{y0}

$$\sin\theta = \frac{v_{y0}}{v_o} \quad \boxed{2}$$

Find t_F

object's horizontal motion

$$v_{x0} = \frac{x_F}{t_F} \quad \boxed{3} \qquad v_{x0}$$

Find v_{x0}

$$\cos\theta = \frac{v_{x0}}{v_o} \quad \boxed{4}$$

4 unknowns, 4 equations

Execute the Plan

$\boxed{4}$ $\cos\theta = \frac{v_{x0}}{v_o}$

$v_o \cos\theta = v_{x0}$ into $\boxed{3}$

$v_o \cos\theta = \frac{x_F}{t_F}$

$t_F = \frac{x_F}{v_o \cos\theta}$ into $\boxed{1}$

$$y_F = -\frac{1}{2}g\left(\frac{x_F}{v_o\cos\theta}\right)^2 + v_{yo}\frac{x_F}{v_o\cos\theta}$$

$\boxed{2}$ $\sin\theta = \frac{v_{y0}}{v_o}$

$v_o \sin\theta = v_{y0}$ into $\boxed{1}$

$$y_F = -\frac{1}{2}g\left(\frac{x_F}{v_o\cos\theta}\right)^2 + v_o\sin\theta\frac{x_F}{v_o\cos\theta}$$

$$y_F = -\frac{1}{2}g\left(\frac{x_F}{v_o\cos\theta}\right)^2 + x_F\tan\theta$$

$$y_F = -\frac{1}{2}g\left(\frac{x_F}{v_o\cos\theta}\right)^2 + x_F\tan\theta$$

This is the equation of a parabola

$$y = ax^2 + bx + c$$

Evaluate the function with specific values

$\theta = 45°$, $v_o = 4m/s$, $g = -10m/s^2$

Graph it

x in m	y in m
0	0
1	.38
2	-.50
1.5	.09
.5	.34
.7	.39
1.2	.29
.3	.24
.8	.40