This Week

Textbook -- Read Chapter 4, 5

Competent Problem Solver - Chapter 4

Pre-lab Computer Quiz

What's on the next Quiz?

Check out sample quiz on web by Thurs.

- What you missed on first quiz
- Kinematics - Everything
  - Check Your Understanding at end of each Laboratory is a good guide
- Forces - to lecture before quiz
- Work problems as if taking the quiz:
  - Assigned homework - minimum if you can do them easily
  - Problems at end of chapter 3 and 4 of CPS
  - Other textbook problems

Theory of Forces

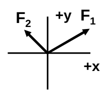
The combined effect of ALL forces on an object that determines its acceleration.

The effects of forces in perpendicular directions are independent

The Mathematics:

- Forces are vector quantities
- Combining Forces means Vector Addition
  - Define a coordinate system with perpendicular x and y axes

Example: 2 forces act on an object, how does it move?



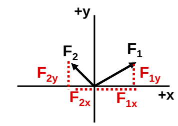
Add x components of forces

and independently

Add y components of forces

Adding Forces

First: Take the vectors apart



$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2$

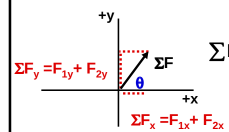
means

$\Sigma F_x = F_{1x} + F_{2x}$

and

$\Sigma F_y = F_{1y} + F_{2y}$

Second: Put the resulting vector together



$$\Sigma F = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

Review

The sum of the x components of Forces

ΣF_x

affects ONLY

the x component of acceleration

ΣF_x causes a_x

The sum of the y components of Forces

ΣF_y

affects ONLY

the y component of acceleration

ΣF_y causes a_y

The sum of the z components of Forces

ΣF_z

affects ONLY

the z component of acceleration

ΣF_z causes a_z

Forces and Motion

The relationship between

Forces on an object and
Acceleration of an object

was found to be amazingly simple

The x component of acceleration of an object

is proportional to

the sum of the x components of all of the forces on that object from the interactions of all other objects.

The constant of proportionality is

The mass of the object

$\Sigma F_x = m\, a_x$

The same is true for the y and z components

Vector Notation

$\Sigma \vec{F} = m\, \vec{a}$

$\Sigma \vec{F}$ is the sum of the interactions of "your" object with all <u>other</u> objects

$\vec{a}$ is the acceleration of "your" object

m is the mass of "your" object

$\Sigma \vec{F} = m\, \vec{a}$ means:

$\Sigma F_x = m\, a_x$

$\Sigma F_y = m\, a_y$

$\Sigma F_z = m\, a_z$

Where x, y, z are three perpendicular directions.

Known as Newton's 2nd Law

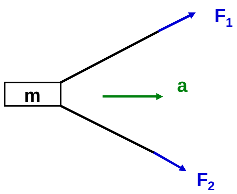
If an object is accelerating

Is there always a force in the direction of the acceleration?

(a) Yes

(b) No

If no, give an example

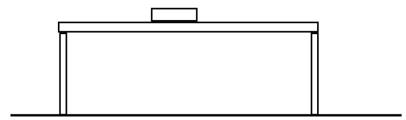


There is no "real" force in the direction of a

Analyze the Forces

Finding all of the forces and their values

Simple case: A book on a table



Draw all relevant forces.

Did you get them all?

How do they combine?

How are they related?

Use diagrams to clarify

Step 1: Isolation

Isolate the object you are interested in

Draw only the forces on that object

Only those forces determine its acceleration

There appear to be 2 important objects here:

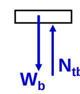
The book

The table

First Object: the book

Free-body Diagram of Book

Only the forces on the book

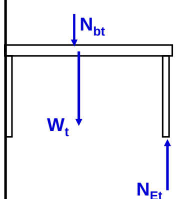


W_b : gravitational pull of the Earth on the book (weight of the book)

N_{tb} : contact push of the table on the book (perpendicular to the table/book contact)

Second Object: the table

Free-body Diagram of Table



W_t : gravitational pull of the Earth on the table (weight of the table)

N_{bt} : contact push of the book on the table (perpendicular to the book/ table contact)

N_{Et} : contact push of the ground on the table (perpendicular to the ground/table contact)

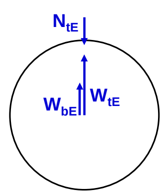
Contact pushes by a surface are usually called normal forces

Normal is an old fashioned word for perpendicular

One more relevant Object: the Earth

Third Object: the Earth

Free-body Diagram of Earth



W_{bE} : gravitational pull of the book on the Earth

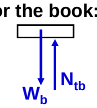
W_{tE} : gravitational pull of the table on the Earth

N_{tE} : contact push of the table on the ground (perpendicular to the ground/table contact)

How are these forces related to each other?

Relationship of Forces on a Single Object

For the book: $\Sigma F_y = m\, a_y = 0$



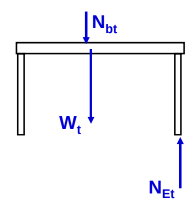
$N_{tb} - W_b = 0$

$N_{tb} = W_b$

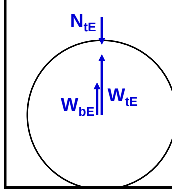
For the table:

$\Sigma F_y = m\, a_y = 0$

$N_{Et} = W_t + N_{bt}$



For the Earth:

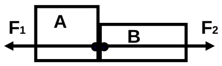


$\Sigma F_y = m\, a_y = 0$

$N_{tE} = W_{bE} + W_{tE}$

Relationship of Forces on Interacting Objects

Object B pushes on Object A



Free-body Diagram of A

Free-body Diagram of B

F_1 is the contact push of object B on object A.

F_2 is the contact push of object A on object B.

F_1 and F_2 are always

equal in magnitude
and
opposite in direction.

$\vec{F_1} = -\vec{F_2}$

Newton's 3rd Law

When any two objects interact

There is ALWAYS a force on each object caused by the other object

These forces are ALWAYS equal in magnitude and opposite in direction

Newton's 3rd Law

These two forces are called 3rd Law Pairs

3rd Law Pairs ALWAYS

Act on DIFFERENT objects

Never show up on the same free body diagram

Are the SAME type of force

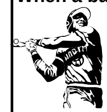
Newton's third law is true

Whether or not the objects are accelerating

Whether or not one object is much larger than the other

Examples

When a baseball player hits a ball with a bat



The force of the bat on the ball is

(a) larger than the force of the ball on the bat
(b) smaller than the force of the ball on the bat
(c) equal to the force of the ball on the bat

When speeding car runs over a rat

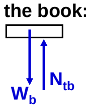
The force of the car on the rat is

(a) larger than the force of the rat on the car
(b) smaller than the force of the rat on the car
(c) equal to the force of the rat on the car

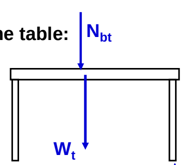
Every force acting on your object has a third law pair that is a force on another object

Relationship of Forces on Different Objects

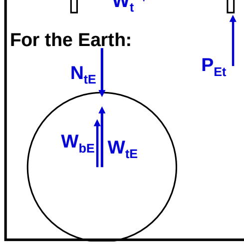
For the book:



For the table:



For the Earth:



Book and table

$N_{tb} = N_{bt}$

Book and Earth

$W_b = W_{bE}$

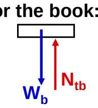
Table and Earth

$W_t = W_{tE}$

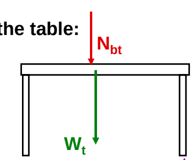
$N_{Et} = N_{tE}$

Summary

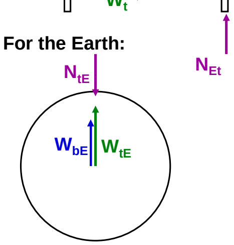
For the book:



For the table:



For the Earth:



From $\Sigma F = ma$

$N_{tb} = W_b$

$N_{Et} = W_t + N_{bt}$

$N_{tE} = W_{bE} + W_{tE}$

From Third Law

$N_{tb} = N_{bt}$

$W_b = W_{bE}$

$W_t = W_{tE}$

$N_{Et} = N_{tE}$

Weight

The weight of an object is the gravitational force on it exerted by a much larger object.

Near the surface of the Earth that other object is the Earth

How do you calculate that force? We will get to that at the end of this semester

For now use Newton's 2nd law and a thought experiment to find what this gravitational force equals

Suppose you drop a book, the force on it is the

Gravitational force of the Earth on the book

$\Sigma F = W_b$

The book accelerates with a value g

$\Sigma F = ma = mg$

Thus $W_b = mg$

True for any object near the surface of the Earth

Example

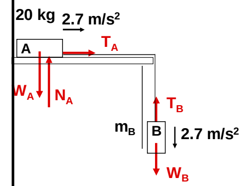
As might be stated in your text

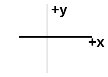
A 20 kg block rests on a frictionless table. A cord attached to the block extends horizontally to a pulley at the edge of the table. When another block of unknown mass is hung at the end of the cord after it passes over the pulley, the hanging block accelerates downward at 2.7 m/s^2 and pulls the other block with it. Calculate the mass of the hanging block and the tension in the cord.

As it might appear on a quiz

You have been hired as a consultant for a new movie about an expedition to the South Pole. In one scene, the explorers are on a glacier that comes to an end with a steep cliff. They need to get their supplies down the cliff. For safety, each box of supplies is roped to another box with a 30 m rope. One box breaks away and falls off the cliff. The 20 kg box it is roped to is pulled over the horizontal ice of the glacier towards the edge of the cliff. It is important that the hero of the story save this box. You calculate that this can be done if this it has an acceleration of 2.7 m/s^2. Now you need to know the mass of the other box and the necessary strength of the rope.

FOCUS THE PROBLEM





You recognize this from the lab

Question: Find mass of hanging block and tension in cord

Approach:

Use forces on B to relate acceleration to m_B.

A and B are tied together so they have the same magnitude of acceleration

Use forces on A to relate acceleration to T_A

For massless rope $T_A = T_B$.

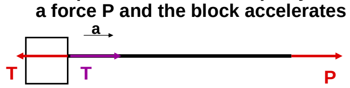
Since B accelerates, force of rope on B is less than its weight

Assumptions: massless cord, frictionless ice

The Massless Rope

A very useful approximation

You pull a block with a rope by exerting a force P and the block accelerates



The rope pulls on the block with a force T

How are P and T related?

The block pulls on the rope with a force T.

3rd Law Pairs

Free body diagram of the rope

T P

$\Sigma F = m_{rope}\, a$

Approximate $m_{rope} = 0$

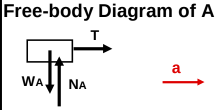
$\Sigma F = 0$

Thus T = P

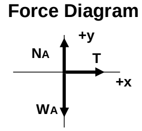
The force exerted on the rope equals the force the rope exerts on the block

Physics Description

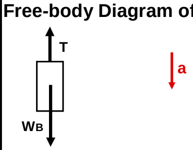
Free-body Diagram of A



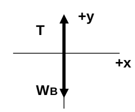
Force Diagram of A



Free-body Diagram of B



Force Diagram of B



$m_A = 20$ kg

$a_B = 2.7\ m/s^2$

Target quantities: m_B and T

Relevant Equations:

$\Sigma F_x = m\, a_x$ $\Sigma F_y = m\, a_y$ $W = mg$

Block A

$T = m_A a$ $N_A - W_A = 0$

Block B

$T - W_B = m_B(-a)$

Plan the Solution

unknowns

Find m_B — m_B

consider the motion of object B.

$T - W_B = m_B(-a)$ [1] — T, W_B

Find W_B

$W_B = m_B\, g$ [2]

$-m_B\, a = T - m_B\, g$

Find T

Must consider object A to get more information

For object A

$T = m_A a$ [3]

Note: 3 unknowns, 3 equations ok

Execute the Plan

$m_A a - m_B g = -m_B a$ Putting [3] and [2] into [1]

$m_A a = m_B g - m_B a$

$m_A a = m_B (g - a)$

$\frac{m_A a}{(g - a)} = m_B$

Check units

accel. units cancel giving mass units, ok

$m_B = \frac{(20 kg)(2.7 m/s^2)}{(9.8 m/s^2 - 2.7 m/s^2)}$

$m_B = 7.6 kg$

Now for the other target, T

Looking at the mathematical plan

equation [3]

$T = m_A a$

$T = (20 kg)(2.7 m/s^2)$

$T = 54 kg\, m/s^2 = 54 N$

Evaluate the Solution

Is it properly stated?

Quantities in the mathematics are described in the picture

Quantities solved for have appropriate units

mass in Kg

force in N

Is it unreasonable?

Since B accelerates downward, $T < W_B$

$W_B = m_B g$

$W_B = 7.6 kg (10 m/s^2)$

$W_B = 76 N$

$54 N < 76 N$ This is reasonable

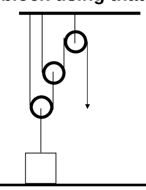
The mass of B (7.6 kg) is reasonable for a small box

Is it complete?

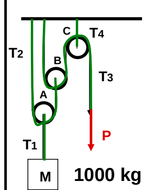
Yes, found both T and m_B

Example

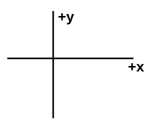
As the engineering advisor to an archeological team, you are trying to figure out how the ancient Egyptians could lift large blocks of stone from a quarry. The team has found evidence of wooden disks which could have been used as pulleys. The team leader has suggested that a three pulley system with one fixed pulley and two movable pulleys might have been used. You to have been assigned determine if the ropes used by the Egyptians would have been strong enough for such a system to lift a 1000 kg block of stone using the pulley system sketched below. You also want to know if one person could lift a block using that system.



Focus



1000 kg



Question: What is the force on each rope?
What is force needed to lift block?

Approach

There are 4 ropes in the problem

Use Newton's 3rd Law to relate force exerted on a rope to force exerted by the rope by other objects.

Objects are: block and 3 pulleys

Assume that block is pulled up at a constant velocity.

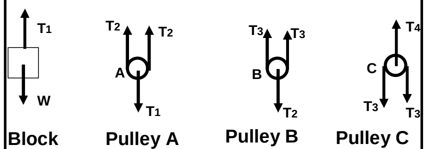
ΣF_y on each pulley is 0 (no acceleration)

Assume massless ropes

Assume frictionless, massless pulleys

Describe the Physics

Free - body diagrams



M = 1000 kg

a = 0

Target variables: T_1, T_2, T_3, T_4

Relevant Equations: $\Sigma F_y = m a_y$

Block: $T_1 - W = 0$

Pulley A: $2T_2 - T_1 = 0$

Pulley B: $2T_3 - T_2 = 0$

Pulley C: $T_4 - 2T_3 = 0$

$W = Mg$

Plan

unknowns

Find T_1 — T_1

consider the motion of the block

$T_1 + (-Mg) = 0$

$T_1 = M g$ [1]

Find T_2 — T_2

consider the motion of pulley A

$2 T_2 - T_1 = 0$

$T_2 = Mg/2$ [2]

Find T_3 — T_3

consider the motion of pulley B

$2T_3 - T_2 = 0$

$T_3 = Mg/4$ [3]

Find T_4 — T_4

consider the motion of pulley C

$T_4 - 2T_3 = 0$

$T_4 = Mg/2$ [4]

Execute

$T_1 = Mg$

$T_1 = (1000\ kg)(9.8\ m/s^2)$

$T_1 = 9800\ N$

$T_2 = Mg/2$

$T_2 = 4900\ N$

$T_3 = Mg/4$

$T_3 = 2450\ N$

$T_4 = Mg/2$

$T_4 = 4900\ N$

Rope attached to the block must be the strongest. It must exert a force at least as large as the weight of the block.

The force required to lift the block is the 3rd Law pair to T_3. It is 2450 N.

Evaluate

The force exerted by each rope is given in Newtons which is a unit of force.

The force exerted by each rope is never greater than the weight of the block so the answers are not unreasonable. The force exerted by a person lifting the block is less than the weight of the block. That is reasonable for a useful machine.

By Newton's 3rd Law, the force exerted by a rope is equal to the force applied to the rope. The force applied to the rope is its minimum strength. The first question is answered.

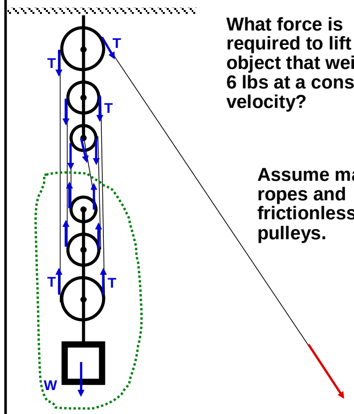
The force that a person needs to apply to lift the block is the 3rd Law Pair of T_3. The second question is answered.

Note that this example is a very useful gadget.

To lift an object with a weight of 9800 Newtons at a constant velocity you only need to exert a force of 2450 Newtons

You have achieved a "mechanical advantage" of a factor of 4 !

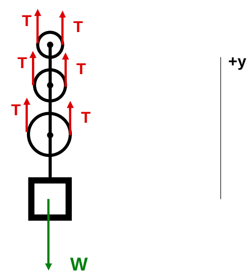
A real pulley design



What force is required to lift an object that weighs 6 lbs at a constant velocity?

Assume massless ropes and frictionless, pulleys.

Free-body Diagram



Motion in the y direction:

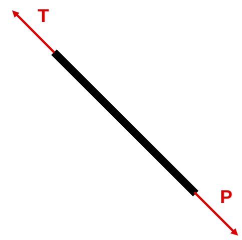
$a_y = 0, \ \Sigma F_y = m a_y = 0$

$6T = W$

$T = W/6$

Rope

Free-body Diagram



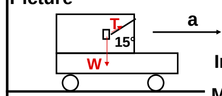
$F_p = T$

Example

Riding in a friend’s sports car you feel the seat pushing on your back when it pulls away from a stop light. You decide you want to know the 1000 kg car’s acceleration. How many g's are you "pulling"? You notice there is a die hanging from the rear view mirror. Very retro. When the car leaves the intersection the die makes an angle of 15 ° with the vertical. Later you measure the mass of that die to be 100 grams. What was the acceleration of the car?

Focus

Picture



Initial velocity of car : 0

Mass of die : 100 grams

Mass of car: 1000 kg

Are we sure that the angle of the die is as drawn?

Question: What is the acceleration of car?

Approach:

Relate acceleration to force on the die

Acceleration of car is acceleration of die

Consider forces on die

By Earth (Weight)

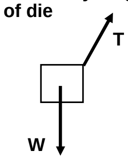
By string (tension)

The acceleration in one direction is independent of the forces in perpendicular directions

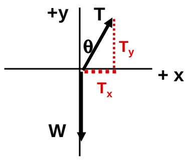
Use components

Physics Description

Free body diagram of die



Force diagram of die



m = 100 gr

$a_y = 0$

target variable: a_x

Relevant equations:

$\Sigma F_x = m\, a_x$ $\quad T_x = ma_x$

$\Sigma F_y = m\, a_y$ $\quad T_y - W = ma_y = 0$

$W = m\, g$

$\sin\theta = \frac{T_x}{T}$ $\quad \cos\theta = \frac{T_y}{T}$ $\quad \tan\theta = \frac{T_x}{T_y}$

Plan

		unknowns
Find a_x		a_x
Horizontal motion of the die		
$T_x = ma_x$	1	T_x
Find T_x		
$\tan\theta = \frac{T_x}{T_y}$	2	T
Find T_y		
Vertical motion of the die		
$T_y - mg = 0$	3	T_y
3 unknowns, 3 equations		done

Execute:

$T_y = mg$

$\tan\theta = \frac{T_x}{mg}$

$mg \tan\theta = T_x$

$mg \tan\theta = ma_x$

$\boxed{g \tan\theta = a_x}$

$a_x = \tan(15°)\, g$

$\boxed{a_x = 0.27\, g}$

Evaluate

a_x is in same units as g which is an acceleration so units are correct

The question is answered since the car is accelerating in the x direction and the die is moving with the car.

Is the answer unreasonable?

A car which goes from 0 to 50 mph in 10 seconds has a high acceleration

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$a_{av} = \frac{(50\ \frac{mi}{hr})}{10\ sec}$$

$$a_{av} = \frac{50\ \frac{mi}{hr}\ \frac{5280\ ft}{mi}\ \frac{1\ hr}{60\ min}\ \frac{1\ min}{60\ sec}}{10\ sec}$$

$$a_{av} = \frac{7\ ft}{sec^2} < 1/4\ g \quad \text{since } g = \frac{32\ ft}{sec^2}$$

This car’s acceleration is about that so the answer is reasonable