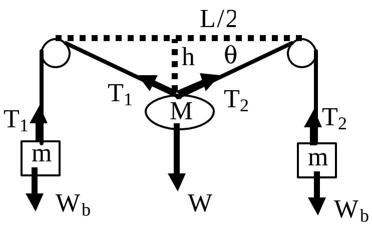
Problem Solutions for Physics 1301 Final

Problem 1

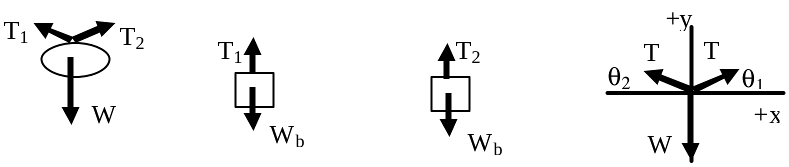


What is h as a function of m, M, and L?

$$\sin\theta = \frac{h}{\sqrt{\left(\frac{L}{2}\right)^2 + h^2}}$$

Approach: Use dynamics, acceleration = 0

Free body diagrams Force diagram



$$T_1 - W_b = 0 \qquad T_2 - W_b = 0$$

$$T_1 = W_b \qquad T_2 = W_b$$

$$T_1 = T_2 = T = W_b = mg$$

$$\sum F_y = T\sin\theta + T\sin\theta - Mg = 0$$

$$\sum F_x = T\cos\theta_1 - T\cos\theta_2 = 0$$

$$\theta_1 = \theta_2 = \theta$$

Target : h(m,M,L)
Assume the frictional force on the pulley is negligible.
Assume the string mass can be neglected.

Plan		unknowns
Find h		h
$\sin\theta = \frac{h}{\sqrt{\left(\frac{L}{2}\right)^2 + h^2}}$	[1]	$\sin\theta$
Find $\sin\theta$		
$T\sin\theta + T\sin\theta - Mg = 0$	[2]	T
Find T		
$T = mg$	[3]	

3 unknowns, 3 equations – ok to solve

Solve [3] for T and put into [2]

$$2mg\sin\theta - Mg = 0$$

solve for $\sin\theta$ and put into [1]

$$\sin\theta = \frac{M}{2m}$$

$$\frac{M}{2m} = \frac{h}{\sqrt{\left(\frac{L}{2}\right)^2 + h^2}}$$

$$\frac{M}{2m}\sqrt{\left(\frac{L}{2}\right)^2 + h^2} = h$$

$$\left(\frac{M}{2m}\right)^2\left(\left(\frac{L}{2}\right)^2 + h^2\right) = h^2$$

$$\left(\frac{M}{2m}\right)^2\left(\frac{L}{2}\right)^2 = h^2 - \left(\frac{M}{2m}\right)^2 h^2$$

$$\boxed{\frac{\left(\frac{M}{2m}\right)\left(\frac{L}{2}\right)}{\sqrt{1-\left(\frac{M}{2m}\right)^2}} = h}$$

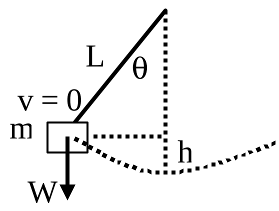
Evaluate:
Check units: This is correct since the sag has units of length and all the mass units cancel.

If the mass of the object (M) is larger than 2m, there is no solution for h. This is reasonable since a heaviest object that can be held by the force caused by the two hanging masses has a mass of 2m.

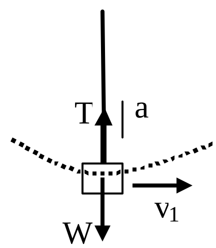
If the distance between pulleys (L) is larger, h must be larger. This is reasonable since a longer string will sag more.

Problem 2

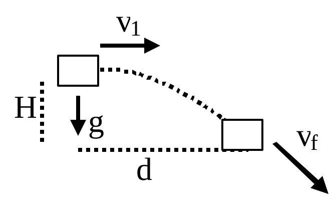
Initial time



time 1 – at bottom of swing



time 2 – hits the ground after flying through the air.



$\theta = 37$o $H = 1.0$ m

$L = 4.0$ m $m = 70$ kg

Find strength of the rope to keep from breaking (T) and distance away it hits the floor if it does break (d).

Approach:

Use dynamics to get the force of the rope on the package at time 1.

$$\sum F_y = T - mg = ma_y$$

Since the package is moving in a circle, at the bottom

$$a_y = \frac{v_1^2}{L}$$

Get the speed of the package at the bottom from conservation of energy.

Use conservation of energy between the initial time and time 1.

$$E_1 - E_o = \Delta E_{transfer}$$

System: package and Earth

Initial time: energy of the system. $E_o = mgh$.

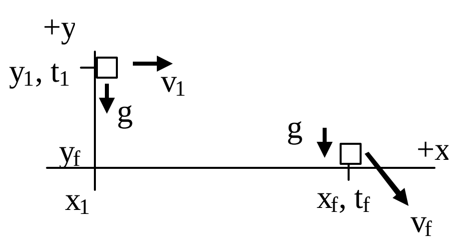
Time 1: energy of the system. $E_1 = \frac{1}{2}mv_1^2$.

$$\Delta E_{transfer} = 0$$

$$mgh - \frac{1}{2}mv_1^2 = 0$$

Use geometry to get h: $\cos\theta = \frac{L-h}{L}$

Use kinematics between time 1 and final time:



$y_1 = H$ $y_f = 0$

$x_1 = 0$ $x_f = d$

$t_1 = 0$ $t_f = ?$

$v_1 = ?$ $v_f = ?$

$a_y = g$

Vertical and horizontal motions are independent

Horizontal: constant velocity

$$v_x = v_{x\,av} = \frac{d}{t_f} = v_1$$

Vertical: constant acceleration

$$a_y = a_{y\,av} = \frac{-v_{fy}}{t_f} = -g$$

$$y_f = 0 = -\frac{1}{2}g(t_f)^2 + H$$

Assume the mass of the rope is small (massless rope).
Assume the air resistance is negligible.

Target: T, d

Plan			unknowns
Find T			T
$T - mg = m\frac{v_1^2}{L}$	[1]	v_1	
Find v_1			
$mgh - \frac{1}{2}mv_1^2 = 0$	[2]		h
Find h			
$\cos\theta = \frac{L-h}{L}$	[3]		

3 unknowns, 3 equations – ok to solve

Solve [3] for h and put into [2]

$$\cos\theta = \frac{L-h}{L}$$

$$L\cos\theta = L - h$$

$$h = L - L\cos\theta$$

$mgL(1-\cos\theta) - \frac{1}{2}mv_1^2 = 0$ solve for v_1^2 and put into [1]

$$2gL(1-\cos\theta) = v_1^2$$

$$T = mg(2(1-\cos\theta)+1)$$

$$\boxed{T = mg(3 - 2\cos\theta)}$$

Check units: units are correct for T since mg equals a force.

$$T = (70kg)\left(9.8\frac{m}{s^2}\right)(3 - 2\cos 37^\circ)$$

$$T = 9.6 \times 10^2\ N$$

Evaluate:

If the mass of the package is larger, then the force on the rope is larger at the bottom. This is reasonable since the rope must exert a larger force than the weight of the package.

If the initial angle of the rope were 0, the cosine would be 1 and the force on the rope would just be mg (the weight of the package). This is reasonable since if the rope did not have any initial angle, it would not swing. The package would just hang straight down on the rope and would not be accelerating.

The value for the strength of the rope is reasonable since it is more than the weight of the package (700 N) but not a great deal more.

The value is reasonable since the force of the rope on the package must be larger than the weight of the package since the package is accelerating in the radial direction at the bottom of the swing.

Now solve for d

Plan		unknowns
Find d		d
$\frac{d}{t_f} = v_1$	[1]	v_1, t_f
Find v_1		
$2gL(1 - \cos\theta) = v_1^2$ from solution for T	[2]	
Find t_f		
$0 = -\frac{1}{2}g(t_f)^2 + H$	[3]	

3 unknowns, 3 equations – ok to solve

Solve [3] for t_f and put into [1]

$$0 = -\frac{1}{2}g(t_f)^2 + H$$

$$\sqrt{\frac{2H}{g}} = t_f$$

$$\frac{d}{\sqrt{\frac{2H}{g}}} = v_1$$

Solve [2] for v_1 and put into [1]

$$\frac{d}{\sqrt{\frac{2H}{g}}} = \sqrt{2gL(1-\cos\theta)} \text{ solve for d}$$

$$d = \sqrt{\frac{2H}{g}}\sqrt{2gL(1-\cos\theta)}$$

$$\boxed{d = 2\sqrt{HL(1-\cos\theta)}}$$

Check units: units are correct for d since square root of length squared equals a length.

$$d = 2\sqrt{(1.0\text{m})(4.0\text{m})(1-\cos 37^\circ)}$$

$$\boxed{d = 1.8\text{m}}$$

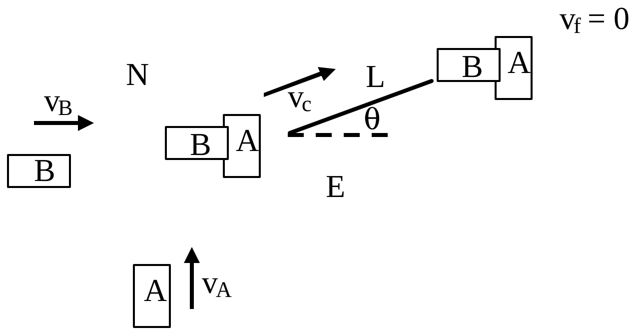
Evaluate:

If the height of the package above the floor at the bottom if its swing were larger, then it will go farther before it hits the floor. This is reasonable since it will have more time to travel before it hits the floor.

If the initial angle of the rope were 0, the cosine would be 1 and it would not go any distance from the bottom of the swing. This is reasonable since if the rope did not have any initial angle, it would not swing. The package would just hang straight down on the rope and not be moving.

The value is reasonable since the horizontal distance is about the same size as the distance off the floor (1m) or the length of the rope (4m).

Problem 3



L = 56 ft
$\theta = 30^o$
$W_A = 2600$ lb
$W_B = 2200$ lb
$\mu = 0.80$

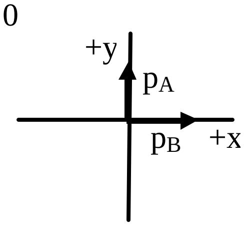
What was the speed of both cars before the collision? Was either speed greater than 50 mph?

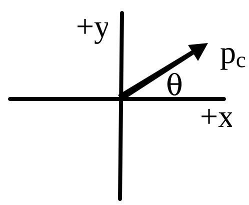
Approach:
Use conservation of momentum in the time interval just before the collision to just after the collision.

System: both cars

$\vec{p}_f - \vec{p}_i = \Delta\vec{p}_{tranfer}$

Assume the time interval of the collision is small so there is a negligible momentum transfer from the frictional force. The weight and normal force are perpendicular to the plane of the street so cannot transfer any momentum in that direction. $\Delta\vec{p}_{tranfer} = 0$





Conservation of momentum:

$p_{fx} - p_{ix} = 0$

$p_{ix} = m_B v_B$

$p_{fx} = (m_A + m_B)v_c \cos\theta$

$p_{fy} - p_{iy} = 0$

$p_{iy} = m_A v_A$

$p_{fy} = (m_A + m_B)v_c \sin\theta$

$W = mg$

$$\boxed{(W_A + W_B)v_c \cos\theta - W_B v_B = 0}$$

$$\boxed{(W_A + W_B)v_c \sin\theta - W_A v_A = 0}$$

Use conservation of energy in the time interval just after the collision until the two cars stop.

System: both cars

$E_f - E_i = \Delta E_{tranfer}$

$$E_i = \frac{1}{2}(m_A + m_B)v_c^2$$

$$E_f = 0$$

The frictional force transfers energy out of the system. The frictional force is constant during the skid.

$$f = \mu F_N$$

The weight and normal force are perpendicular to the motion of the cars so cannot transfer any energy.

$$\Delta E_{tranfer} = -\int_{path} f\,d\ell = -\mu F_N L$$

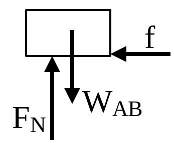
$$0 - \frac{1}{2}(m_A + m_B)v_c^2 = -\mu F_N L$$

$$\boxed{\frac{1}{2g}(W_A + W_B)v_c^2 = \mu F_N L}$$

Get the normal force on the cars from dynamics. There is no acceleration perpendicular to the road.

$$\sum F_z = F_N - W_{AB} = 0$$

$$\boxed{F_N - (W_A + W_B) = 0}$$



Plan		unknowns
Find v_A		v_A
$(W_A + W_B)v_c \sin\theta - W_A v_A = 0$	[1]	v_c
Find v_c		
$\frac{1}{2g}(W_A + W_B)v_c^2 = \mu F_N L$	[2]	F_N
Find F_N		
$F_N - (W_A + W_B) = 0$		

3 unknowns, 3 equations – ok to solve

Solve [3] for F_N and put into [2]

$$F_N = (W_A + W_B)g$$

$$\frac{1}{2g}(W_A + W_B)v_c^2 = \mu(W_A + W_B)L$$

Solve [2] for v_c and put into [1]

$$v_c = \sqrt{2\mu g L}$$

$$(W_A + W_B)\sqrt{2\mu g L}\sin\theta - W_A v_A = 0$$ solve for v_A

$$\boxed{\frac{(W_A + W_B)}{W_A}\sqrt{2\mu g L}\sin\theta = v_A}$$

Check units: $\sqrt{\left[\frac{m}{s^2}\right][m]} = \left[\frac{m}{s}\right] = [v]$ This is correct since m/s is a units of speed.

$$\frac{(2600\text{lb} + 2200\text{lb})}{2600\text{lb}}\sqrt{2(0.80)\left(32\frac{\text{ft}}{\text{s}^2}\right)(56\text{ft})}\sin 30^\circ = v_A$$

$$49.4\frac{\text{ft}}{\text{s}}\left(\frac{1\text{mi}}{5280\text{ft}}\right)\left(\frac{60\text{s}}{1\text{min}}\right)\left(\frac{60\text{ min}}{1\text{hr}}\right) = v_A$$

$\boxed{34\frac{\text{mi}}{\text{hr}} = v_A}$ not speeding

Evaluate: 34 mph is a normal speed for driving a car in the city.

Now get the speed of the other car

Plan		unknowns
Find v_B		v_B
$(W_A + W_B)v_c \cos\theta - W_B v_B = 0$	[1]	v_c
Find v_c		
$v_c = \sqrt{2\mu gL}$ from solution for v_A	[2]	
2 unknowns, 2 equations – ok to solve		

Solve [2] for v_c and put into [1]

$(W_A + W_B)\sqrt{2\mu gL}\cos\theta - W_B v_B = 0$ solve for v_B

$$\boxed{\frac{(W_A + W_B)}{W_B}\sqrt{2\mu gL}\cos\theta = v_B}$$

Check units: $\sqrt{\left[\frac{m}{s^2}\right][m]} = \left[\frac{m}{s}\right] = [v]$ This is correct since m/s is a units of speed.

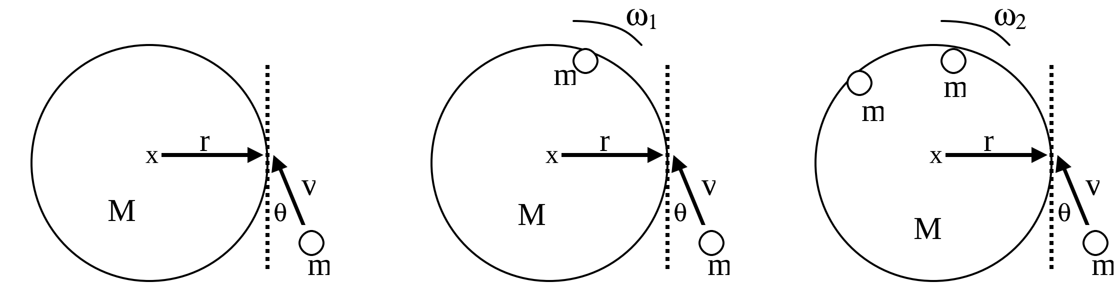
$$\frac{(2600\text{lb} + 2200\text{lb})}{2200\text{lb}}\sqrt{2(0.80)\left(32\frac{\text{ft}}{\text{s}^2}\right)(56\text{ft})}\cos 30^\circ = v_A$$

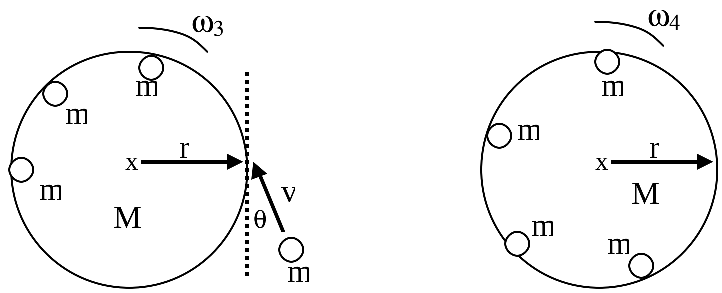
$$101\frac{\text{ft}}{\text{s}}\left(\frac{1\text{mi}}{5280\text{ft}}\right)\left(\frac{60\text{s}}{1\text{min}}\right)\left(\frac{60\text{ min}}{1\text{hr}}\right) = v_A$$

$\boxed{69\frac{\text{mi}}{\text{hr}} = v_B}$ speeding

Evaluate: 69 mph is a normal speed for driving a car on the freeway. It is possible.

Problem 4





W = 50 lb
W_m = 250 lb
r = 9 ft
v = 15 ft/s
$\theta = 20^o$
$I_m = ½ Mr^2$
$I_c = mr^2$

What is the period of the merry-go-round after all the children are on?

Approach:

Use rotational kinematics:

After all the children are on the angular velocity of the merry-go-round is constant.

Neglect friction at the axis of rotation.

Instantaneous angular velocity equals average angular velocity.

$$\omega_4 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

Use conservation of angular momentum to get angular velocity.

$$\vec{L}_f - \vec{L}_i = \Delta\vec{L}_{transfer}$$

Break the problem down to each time interval when a child jumps on.

System: merry-go-round (with any children riding on it) and the child jumping on.

Initial time: Just before jumping on

Final time: Just after jumping on

Time interval 1:	Time interval 2:	Time interval 3:	Time interval 4:
$Li = rmv_t = rmv\cos\theta$	$Li = I_1\omega_1 + rmv\cos\theta$	$Li = I_2\omega_2 + rmv\cos\theta$	$Li = I_3\omega_3 + rmv\cos\theta$
$L_1 = I_1\omega_1$	$L_2 = I_2\omega_2$	$L_3 = I_3\omega_3$	$L_f = I_4\omega_4$
$I_1 = mr^2 + ½ Mr^2$	$I_2 = 2mr^2 + ½ Mr^2$	$I_3 = 3mr^2 + ½ Mr^2$	$I_4 = 4mr^2 + ½ Mr^2$

Conservation of angular momentum for the 4 time intervals:

$$I_1\omega_1 - mv\cos\theta = 0$$

$$I_2\omega_2 - (I_1\omega_1 + mv\cos\theta) = 0$$

$$I_3\omega_3 - (I_2\omega_2 + mv\cos\theta) = 0$$

$$I_4\omega_4 - (I_3\omega_3 + mv\cos\theta) = 0$$

W = mg

Target: T

Plan	unknowns
Find T	T
$\omega_4 = \frac{2\pi}{T}$ [1]	ω_4
Find ω_4	
$I_4\omega_4 - \left(I_3\omega_3 + r\frac{W}{g}v\cos\theta\right) = 0$ [2]	$I_3\omega_3$, I_4
Find I_4	
$I_4 = 4(W/g)r^2 + ½ (W_m/g)r^2$ [3]	
Find $I_3\omega_3$	
$I_3\omega_3 - \left(I_2\omega_2 + r\frac{W}{g}v\cos\theta\right) = 0$ [4]	$I_2\omega_2$
Find $I_2\omega_2$	
$I_2\omega_2 - \left(I_1\omega_1 + r\frac{W}{g}v\cos\theta\right) = 0$ [5]	$I_1\omega_1$
Find $I_1\omega_1$	
$I_1\omega_1 - r\frac{W}{g}v\cos\theta = 0$ [6]	

6 unknowns, 6 equations – ok to solve

Solve [6] for $I_1\omega_1$ and put into [5]

$$I_1\omega_1 = r\frac{W}{g}v\cos\theta$$

$$I_2\omega_2 - \left(r\frac{W}{g}v\cos\theta + r\frac{W}{g}v\cos\theta\right) = 0$$

$$I_2\omega_2 = 2r\frac{W}{g}v\cos\theta \quad \text{put into [4]}$$

$$I_3\omega_3 - \left(2r\frac{W}{g}v\cos\theta + r\frac{W}{g}v\cos\theta\right) = 0$$

$$I_3\omega_3 = 3r\frac{W}{g}v\cos\theta \quad \text{put into [2]}$$

$$I_4\omega_4 - \left(3r\frac{W}{g}v\cos\theta + r\frac{W}{g}v\cos\theta\right) = 0$$

$$I_4\omega_4 = 4r\frac{W}{g}v\cos\theta$$

Solve [3] for I_4 and put into [2] and solve for ω_4

$$\left(4\frac{W}{g}r^2 + \frac{1}{2}\frac{W_m}{g}r^2\right)\omega_4 = 4r\frac{W}{g}v\cos\theta$$

$$\omega_4 = \frac{4\frac{W}{g}v\cos\theta}{4\frac{W}{g}r + \frac{1}{2}\frac{W_m}{g}r} \quad \text{put into [1] and solve for T}$$

$$\frac{4Wv\cos\theta}{4Wr + \frac{1}{2}W_m r} = \frac{2\pi}{T}$$

$$\boxed{T = \frac{\pi r\left(4W + \frac{1}{2}W_m\right)}{2Wv\cos\theta}}$$

Check units: $\frac{[m][N]}{[N]\left[\frac{m}{s}\right]} = [s] = [t]$ This is correct since s is a unit of period.

$$T = \frac{\pi(9ft)\left(4(50lb) + \frac{1}{2}(250lb)\right)}{2(50lb)\left(15\frac{ft}{s}\right)\cos 20^\circ}$$

$$\boxed{T = 6.5s}$$

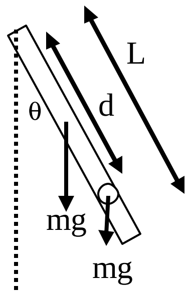
Evaluate:

The period increases if the radius of the merry-go-round increases. That is reasonable since then it would have a larger moment of inertia.

The period decreases if the children run faster (v). That is reasonable since the merry-go-round would rotate faster and it would take less time to go around.

6.5 seconds to go around means the children are traveling at a speed of $2\pi r/T$ or 9ft/s. This is slower than they were running so it is reasonable.

Problem 5



I_{bar} = 1/3 mL^2
I_{ball} = md^2

L = 1.40 m
T= 2 s

Where must the ball be placed to make the period 2.0 s?

Approach:
Use dynamics.

$$\sum \tau = I\alpha$$

$$-\left(\frac{L}{2}mg\sin\theta + dmg\sin\theta\right) = \left(\frac{1}{3}mL^2 + md^2\right)\frac{d^2\theta}{dt^2}$$

assume the pendulum is displaced by a small angle: sinθ = θ

$$-\left(\frac{L}{2}mg + dmg\right)\theta = \left(\frac{1}{3}mL^2 + md^2\right)\frac{d^2\theta}{dt^2}$$

try solution: $\theta = A\cos(2\pi ft + \phi)$

$$\frac{d\theta}{dt} = -A\sin(2\pi ft + \phi)2\pi f$$

$$\frac{d^2\theta}{dt^2} = -A\cos(2\pi ft + \phi)(2\pi f)^2$$

$$-\left(\frac{L}{2}mg + dmg\right)A\cos(2\pi ft + \phi) = -\left(\frac{1}{3}mL^2 + md^2\right)A\cos(2\pi ft + \phi)(2\pi f)^2$$

$$\left(\frac{L}{2}mg + dmg\right) = \left(\frac{1}{3}mL^2 + md^2\right)(2\pi f)^2$$

$$f = \frac{1}{T}$$

$$\boxed{0 = \frac{4\pi^2}{T^2}d^2 - dg + \left(\frac{4\pi^2}{3T^2}L^2 - \frac{L}{2}g\right)}$$

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = \frac{4\pi^2}{T^2},\ b = -g,\ c = \left(\frac{4\pi^2}{3T^2}L^2 - \frac{L}{2}g\right)$$

check units:

$$\frac{\left[\frac{m}{s^2}\right]+\sqrt{\left[\frac{m}{s^2}\right]^2+\left[\frac{1}{s^2}\right]\left[\frac{m^2}{s^2}\right]}}{\left[\frac{1}{s^2}\right]}=\frac{\left[\frac{m}{s^2}\right]}{\left[\frac{1}{s^2}\right]}=[m]=[d] \text{ correct units for distance}$$

$$a=\frac{4\pi^2}{(2s)^2}=\frac{9.9}{s^2}\ ,\ b=-9.8\frac{m}{s^2}\ ,\ c=\left(\frac{4\pi^2}{3(2s)^2}(1.4m)^2-\frac{(1.4m)}{2}\left(9.8\frac{m}{s^2}\right)\right)=-.41\frac{m^2}{s^2}$$

$$d=\frac{-\left(-9.8\frac{m}{s^2}\right)\pm\sqrt{\left(-9.8\frac{m}{s^2}\right)^2-4\left(\frac{9.9}{s^2}\right)\left(-.41\frac{m^2}{s^2}\right)}}{2\left(\frac{9.9}{s^2}\right)}=\frac{9.8\pm 10.6}{19.8}m$$

$$\boxed{d=1.0m}$$

This is not unreasonable since d is less than the length of the bar.