Energy Transfer

How to tell if an external interaction results in an energy input or energy output.

If the external force is in same direction as displacement

Energy input

If the external force is in opposite direction as displacement

Energy output

What if the Force is not in the same line as the displacement?

Examples:

- Sliding down a ramp
- Projectile motion
- Circular motion
- Pendulum

Use appropriate components

Energy Transfer - Example

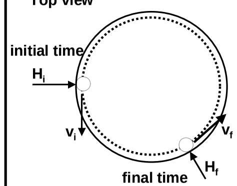
Circular motion on a horizontal table

Ball rolling inside a hoop

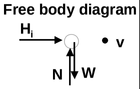
system: ball

Neglect friction

Top view



Free body diagram



Side view

Conservation of energy:

$E_f - E_i = E_{input} - E_{output}$

$E_i = KE_i$

$E_f = KE_f$

$E_{input} = ?$

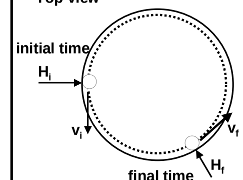
$E_{output} = ?$

Examine each force

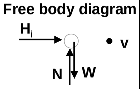
system: ball

Neglect friction

Top view



Free body diagram



Side view

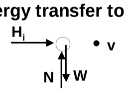
Energy transfer:

Does W ever have a component in the direction of the displacement of the system?

Does N ever have a component in the direction of the displacement of the system?

Does H ever have a component in the direction of the displacement of the system?

In this case an external force does not result in an energy transfer to the system



No force ever has a component in the direction of the ball's displacement

$E_{input} = 0$ $E_{output} = 0$

Conservation of energy:

$E_f - E_i = E_{input} - E_{output}$

$KE_f - KE_i = 0$

$KE_f = KE_i$

$v_f = v_i$ Ball's speed doesn't change

Forces on ball cause it to accelerate

Ball's velocity changes

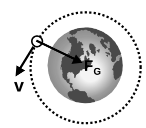
Those forces can't transfer energy to the ball

Ball's speed doesn't change

Orbits

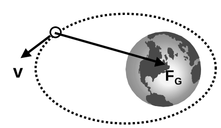
1. If a satellite has a circular orbit around the Earth, does it's speed change?

Why?



2. If a satellite has an elliptical orbit around the Earth, does it's speed change?

Why?



Energy Theory

The energy of every system is always conserved

$E_f - E_i = E_{input} - E_{output}$

If a system is really a single object, the only energy it can have is kinetic energy

$$KE = \frac{1}{2}mv^2$$

Only the component of an external force parallel to an system's direction of motion can transfer energy to the system.

The component of an external force in the same direction as the motion of an object inputs energy to the system.

$$E_{input} = \int_{x_i}^{x_f} F_x dx$$

The component of an external force in the opposite direction as the motion of an object outputs energy from the system.

$$E_{output} = \int_{x_i}^{x_f} F_x dx$$

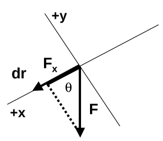
Using Vector Mathematics

ENERGY TRANSFER

Take the component of the force along the displacement and integrate (add up)

$$\int \mathbf{F_r dr}$$

Choose coordinate system so that one axis is along the displacement (direction of motion).



$$E_{in} = \int_{x_i}^{x_f} F_x dx$$

$\cos\theta = F_x / F$

Component of F along dr direction, F_x :

$F_x = F \cos\theta$

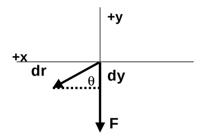
F_r dr = F cosθ dr   dr = dx

$$E_{in} = \int_{x_i}^{x_f} F \cos\theta dr = \int_{x_i}^{x_f} F \cos\theta dx$$

But what if you want to use a different coordinate system?

Energy Transfer - Another Way

Suppose you want to have the coordinate system with one axis along the external force.



cosθ = dy/ dr

Find the component of dr along the F direction, dy :

dy = dr cosθ

Calculate the force times the component of displacement along the force

F dy = F dr cosθ   = F cosθ dr   = F_r dr

same as component of force along the displacement

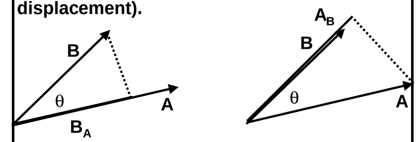
$$E_{transfer} = \int F_r dr = \int F dr_F$$

Dot Product

Both ways are written mathematically as the scalar or "dot" product of 2 vectors

$$\vec{A} \bullet \vec{B}$$

For any two vectors (for example force and displacement).



A times the component of B along A: (A B_A)

A (B cosθ)

equals

B times the component of A along B: (B A_B)

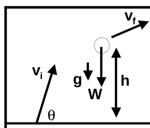
B (A cosθ)

Yes!! A (B cosθ) = B (A cosθ)

Since energy transfer can be calculated either way, we can write it $\int \vec{F} \bullet d\vec{r}$

Example

o help deliver supplies to flood victims in emote locations, you have been asked to nvestigate the feasibility of putting food in a trong container and firing it from a cannon. t the top of its trajectory, a parachute would pen and the container would drift to the round. To determine the corrections ecessary due to air resistance, you first ecide to calculate the container's speed as a unction of its height on its way toward its ighest point and its initial speed without aking the interaction with the air into account.



know	want
v_i	v_f
h	

The motion is completely determined by the initial velocity [magnitude (given) and direction (not given)]

Using the definitions of velocity and acceleration you can calculate everything.

Do it

The only force on the object is the gravitational force, W

Constant, down

Acceleration is constant, down

Horizontal component of velocity is constant

Is it easier using conservation of energy?

Want to find speed

Information is about initial speed

No information about direction of velocity

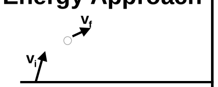
No information about time

Conservation of Energy Approach

System: projectile

Initial time: Launch

Final time: At height h



Conservation of energy equation:

$E_f - E_i = E_{in} - E_{out}$

$E_i = (1/2)\ m\ v_i^2$

$E_f = (1/2)\ m\ v_f^2$

E_{in} = ?

E_{out} = ?

Gravitational force has a component in the opposite direction from the projectile's displacement.

There is an energy output

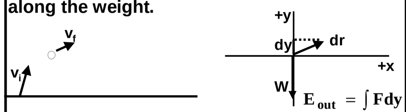
$$E_{out} = \int F_r dr$$

It seems hard to find the component of the weight along the displacement direction because that direction is always changing

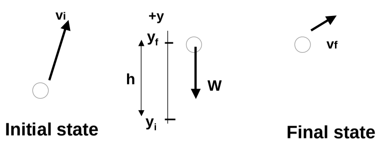
The weight is always in the same direction (vertical) so it seems easier to find the component of the displacement along the weight.

$$E_{out} = \int F dr_F$$

Choose a coordinate system with one axis along the weight.



To use an energy approach we need an Energy diagram



Initial state | Energy transfer | Final state

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target quantity: v_f (as a function of v_i and h)

Quantitative relationships:

Conservation of energy: (system: projectile)

$E_f - E_i = E_{in} - E_{out}$

$$E_i = KE_i = \frac{1}{2}mv_i^2$$

$$E_f = KE_f = \frac{1}{2}mv_f^2$$

$E_{in} = 0$

$$E_{out} = \int_{y_i}^{y_f} W dy$$

W = mg

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \int_{y_i}^{y_f} mg\,dy$$

Plan: | unknown
Find v_f | v_f

conservation of energy

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \int_{y_i}^{y_f} mg\,dy \qquad m$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mg\int_{y_i}^{y_f} dy$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mg(y_f - y_i)$$

Since every term in the conservation of energy equation depends on m, it cancels out of the problem

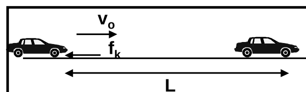
$$\frac{1}{2}v_f^2 - \frac{1}{2}v_i^2 = -g(h)$$

$$v_f^2 - v_i^2 = -2gh$$

$$v_f = \sqrt{v_i^2 - 2gh}$$

Example

The company you are working for has just received a contract to provide the police department with personal data assistants. As part of the functionality, you have been told to write a computer program that will calculate a car's speed from the length of its skid marks and the coefficient of kinetic friction between the tires and the road. All the policeman has to do is punch in the make of the tires and their tread condition, the type of road and driving conditions, and the length of the skid marks. Your formula should do the rest.



What was the car's initial speed?

Use conservation of energy.

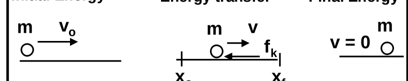
Object is the car.

Energy transferred out of the system by the frictional force.

Get frictional force from normal force

Use vertical forces to get normal force

Initial Energy | Energy transfer | Final Energy

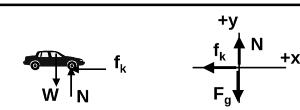


m = ?
v_o = ?

$x_o = 0$
$x_f = L$

$E_i = \frac{1}{2}mv_0^2$ | $E_{out} = \int f_k dx$ | $E_f = 0$

$E_f - E_i = E_{in} - E_{out}$



$f_k = \mu_k N$ | $\sum F_y = ma_y$

$N - mg = 0$

Target: v_o = ?

unknowns

Find v_o | v_o

$$0 - \frac{1}{2}mv_0^2 = 0 - \int f_k dx$$

$$\frac{1}{2}mv_0^2 = f_k\int_0^L dx = f_k L \quad \boxed{1} \qquad f_k \quad m$$

Find f_k

$f_k = \mu_k N$ $\boxed{2}$ N

Find N

$N - mg = 0$ $\boxed{3}$

4 unknowns, 3 equations -- mass will cancel

$$N - mg = 0$$

$$N = mg$$

$$f_k = \mu_k mg$$

$$\frac{1}{2} m v_0^2 = \mu_k mgL$$

$$\frac{1}{2} v_0^2 = \mu_k gL$$

$$\boxed{v_o = \sqrt{2\mu_k gL}}$$

check units

$$\sqrt{\left[\frac{m}{s^2}\right][m]} = \left[\frac{m}{s}\right]$$

Example - Chap. 8, 75 (a)

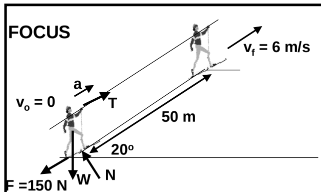
Now your manager tells you that the police personal data assistant must calculate the car's speed from the skid marks if the car is going down a hill. The police officer will need to input the angle of the hill.

Try it!

Example - Problem 69 - Chap. 6

You have a job providing the engineering help for an architect in Colorado. You are currently designing a cable tow to pull skiers up a hill so they can ski down. The customer would like the cable tow to pull a skier uphill at constant acceleration from the bottom reaching a speed of 6 m/s at the top. You need to determine what type of cable you should purchase. The hill is 50 m long and inclined at 20 degrees from the horizontal. By measuring skier speeds on a downhill run, you know there is a friction force of 150 N between the skis and the snow independent of the skier's weight.

FOCUS



Question: What force must the cable withstand?

Approach:

Use conservation of energy to relate the final speed and the forces.

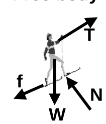
The rope force (T), the frictional force (f), and the weight (W) all have components in the direction of the displacement.

Energy transfer occurs

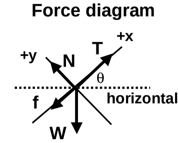
Use a coordinate system with one axis along the ski slope

PHYSICS DESCRIPTION

Free body diagram

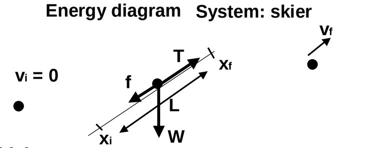


Force diagram



Energy diagram

System: skier

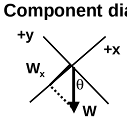


initial state

Energy transfer

final state

Component diagram



$\sin\theta = \frac{W_x}{W}$

θ = 20°

$v_f = 6$ m/s

$v_i = 0$

L = 50 m

f = 150 N

Target quantity: T

Quantitative relationships:

Conservation of Energy

$E_f - E_i = E_{input} - E_{output}$

Skier energy is kinetic

$KE = \frac{1}{2} m v^2$

Energy transferred to and from skier

$$E_{transfer} = \int_{x_i}^{x_f} \vec{F} \bullet d\vec{r}$$

$$E_{transfer} = \int_{x_i}^{x_f} F_x dx$$

$$E_{input} = \int_0^L T dx$$

$$E_{output} = \int_0^L (f + W_x) dx$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \int_0^L T dx - \int_0^L (f + W \sin\theta) dx$$

Plan:

unknowns

Find T — T

Conservation of energy

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \int_0^L T dx - \int_0^L (f + W \sin\theta) dx$$

$$\frac{1}{2}mv_f^2 = T\int_0^L dx - (f + mg\sin\theta)\int_0^L dx$$

$$\frac{1}{2}mv_f^2 = TL - (f + mg\sin\theta)L \qquad m$$

1 equation, 2 unknowns

m does not cancel

No other useful information or physics relationships

Need to estimate a maximum mass for a skier

$$\frac{mv_f^2}{2L} + (f + mg\sin\theta) = T$$

$$\frac{mv_f^2}{2L} + (f + mg\sin\theta) = T$$

Check units

T] = [force] , [f] = [force] , [mg sin θ] = [force]

$[\frac{1}{2L} m v_f^2] = [\text{mass}] \frac{[m^2/s^2]}{[m]}$

= [mass] [acceleration]

= [force]

ok all units are force units

Estimate 100 kg for a skier

$T = \frac{1}{2(50\text{ m})}(100\text{ kg})(6\text{ m/s})^2$

$+ (100\text{ kg})(9.8\text{ m/s}^2)\sin 20° + (150\text{ N})$

36 N + 335 N + 150 N = T

T = 521 N

Evaluate:

The force of the rope on the skier (T) will be less than if the skier were just hanging on the rope straight down.

T (est) < W

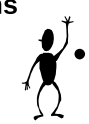
T (est) < mg

T (est) < 100 kg (10 m/s^2)

T less than 1000 N

OK the answer is reasonable

Complex Systems



More than one object

Start with a familiar situation

Drop a ball

Take system as the ball and the Earth

No external force on system

No energy transfer

The ball speeds up

Kinetic energy of the system increases

But the energy of the system cannot increase.

The system initially had unrecognized energy

E_{system} = kinetic energy + new energy

E_{system} = KE + NE

The two object system must give the same result for motion as the one object system

The way objects move doesn't depend on how we choose a system

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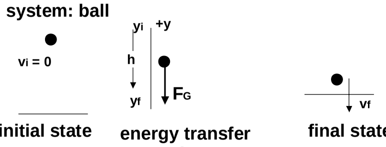
Single object system

F_G is an external force

system: ball



$$E_{input} = \int_0^h F_G dy$$

Conservation of Energy:

$E_f - E_i = E_{in} - E_{out}$

Identify Energy Terms

$E_i = (1/2)\, m v_i^2 = 0$ — $E_{in} = F_G\, h$

$E_f = (1/2)\, m v_f^2$ — $E_{out} = 0$

Conservation of Energy equation:

$(1/2)mv_f^2 - 0 = F_G\, h - 0$

$(1/2)mv_f^2 = mg\, h$

$v_f^2 = 2g\, h$

Two object system

system: ball + Earth

Conservation of Energy:

$E_f - E_i = E_{in} - E_{out}$

Identify Energy Terms

$E_i = (1/2)\, m v_i^2 + NE_i$ — $E_{in} = 0$

$E_f = (1/2)\, m v_f^2 + NE_f$ — $E_{out} = 0$

Conservation of Energy equation:

$(1/2)mv_f^2 + NE_f - [(1/2)mv_i^2 + NE_i] = 0 - 0$

$(1/2)mv_f^2 = NE_i - NE_f$

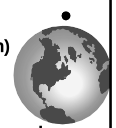
$v_f^2 = (2/m)(NE_i - NE_f)$

Compare Results

One object system (ball)

$v_f^2 = 2g\,h$

Two object system (ball and Earth)



$v_f^2 = (2/m)(NE_i - NE_f)$

ince the ball's speed does not depend on our hoice of system

$2g\,h = (2/m)(NE_i - NE_f)$

$mg\,(y_i - y_f) = (NE_i - NE_f)$

$mg\,y_i - mg\,y_f = NE_i - NE_f$

$mg\,y_i = NE_i$

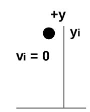
$mg\,y_f = NE_f$

The New Energy depends on the gravitational orce and the vertical position of an object rom the surface of the Earth

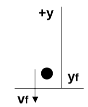
Gravitational Potential Energy

The new energy depends on the relative position of the two objects in the system

Gravitational Potential Energy GPE



no
energy transfer



initial state — final state

GPE depends on the height of an object

The higher an object, the greater its gravitational potential energy

Energy of the ball - Earth system

Throwing a ball straight up

No energy transferred to or from system (neglect air interaction)

Energy of system changes from Kinetic energy to Gravitational Potential energy

Summary

If you choose a system that includes the Earth

GPE must be included in the initial and final energy of a system.

Gravitational force is not an external force

The gravitational force cannot transfer energy into or out of the system.

If you choose a system that does not include the Earth

There is no GPE in your system

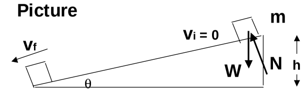
Gravitational force is an external force

The gravitational force can transfer energy into or out of the system.

Example

ou have been asked to design an efficient ackage handling system for a company that ells its products over the web. The assembly epartment is in the basement of a building nd needs to receive large boxes from its uppliers. One of your team members has uggested a ramp with a surface of small ollers so that boxes slip easily down. You ant to make sure that the boxes are not oving too fast for safe handling at the ottom of the ramp. You decide to calculate heir speed as a function of the box's mass, he height of the ramp, and its angle to the orizontal.

Picture



Question: Find the speed of the box at bottom of ramp as a function of m, h, and θ.

Approach: Use conservation of energy

System: box + Earth

Initial time box at top of ramp

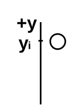
Final time box at bottom of ramp

Contact force with ramp (normal force) gives no energy transfer because it is perpendicular to the velocity

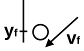
Gravitational force gives no energy transfer because it is within the system

Assume frictionless surface and box starts from rest.

Energy diagram



$v_i = 0$



initial state: $y_i = h$

final state: $y_f = 0$

$E_i = (KE)_i + (GPE)_i$ — $E_f = (KE)_f + (GPE)_f$

$E_i = (1/2)\,m\,v_i^2 + mgy_i$ — $E_f = (1/2)\,m\,v_f^2 + mgy_f$

$E_i = 0 + mgh$ — $E_f = (1/2)\,m\,v_f^2 + 0$

$E_{input} = 0$

$E_{output} = 0$

target quantity: v_f

Quantitative relationships:

$E_f - E_i = E_{in} - E_{out}$

$(1/2)\,m\,v_f^2 - mgh = 0$

Plan: **unknowns**

Find v_f v_f

Conservation of energy

$(1/2)\ m\ v_f^2 - mgh = 0$

$(1/2)\ v_f^2 - gh = 0$

1 equation, 1 unknowns

$v_f = \sqrt{2(gh)}$

Evaluation:

This is the same final speed as if the box had just been dropped straight down

Does the box have the same acceleration?

Dropping straight down, $a_d = g$

Down an inclined plane (ramp)

NO

It takes a longer time to get down the ramp than to drop straight down

Potential Energy

Whenever you have an external force on an object that gives an energy transfer to that object which depends only on

the position change of the object
NOT on the actual path traveled

you can include the object exerting the external force in your system and define an appropriate potential energy for the system

ou can tell if you have this kind of force if hen object returns to the same position

Total energy transferred to it is 0

$\Delta E_{transfer} = 0$

$E_{in} - E_{out} = 0$

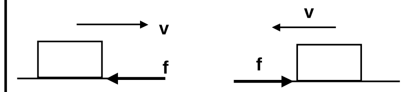
$E_{transfer} = \int_i^f \vec{F} \bullet d\vec{r}$

For which interactions can you define a potential energy?

Gravitational	Electric
Frictional	Spring

FRICTIONAL FORCE

Depends on the path



Because force changes direction when the velocity changes direction.

E_{output} **only**

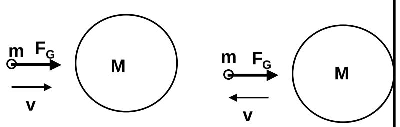
GRAVITATIONAL FORCE

Independent of path

Because force is independent of velocity

E_{output} **or** E_{input}

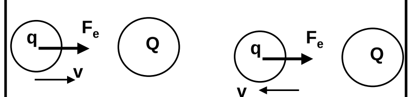
Gravitational Force - Larger scale



E_{output} **or** E_{input}

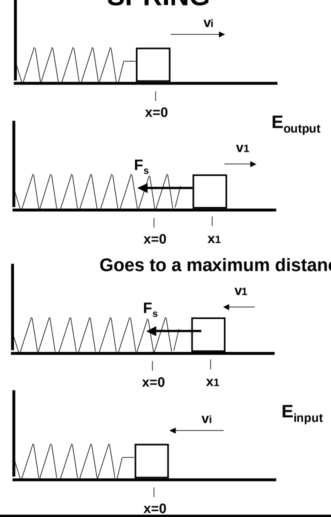
ELECTRIC FORCE

Independent of path



E_{output} **or** E_{input}

SPRING



Energy Transfer

$$E_{out} = \left|\int_0^L kxdx\right| = k\left|\int_0^L xdx\right|$$

$$E_{out} = \frac{1}{2}kL^2$$

$$E_{in} = \left|\int_L^0 kxdx\right| = k\left|\int_L^0 xdx\right|$$

$$E_{in} = \frac{1}{2}kL^2$$

For coming back to the same place

$E_{transfer} = E_{in} - E_{out} = 0$

Or we could have written the energy transfer

$$E_{transfer} = \int_0^0 kxdx = k\int_0^L xdx + k\int_L^0 xdx = 0$$

Energy transferred to the object by the spring depends only on the object's displacement

Does not depend on the actual path

For a round trip, energy transfer = 0

Can invent a potential energy

For the object + spring system

Spring Potential Energy

The motion of the object does not depend on how we choose our system

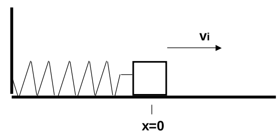
Compare results using conservation of energy

- System : object
  - Only kinetic energy
  - Spring transfers energy

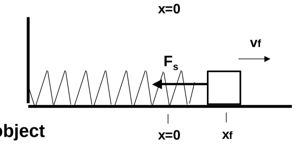
- System : object + spring
  - Kinetic and Potential energy
  - No energy transfer by spring

Single Object System

initial time



final time



System: object

Conservation of Energy

$E_f - E_i = E_{input} - E_{output}$

Identify energy terms:

initial time: $E_i = KE_i = (1/2)\, mv_i^2$

final time: $E_f = KE_f = (1/2)mv_f^2$

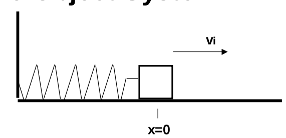
transfer: $E_{out} = \int kx\,dx = k\int x\,dx = \frac{1}{2}kx^2$

$E_{input} = 0$

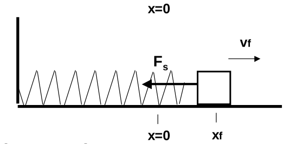
$(1/2)mv_f^2 - (1/2)mv_i^2 = 0 - (1/2)kx_f^2$

Two Object System

initial time



final time



System: object + spring

Conservation of Energy

$E_f - E_i = E_{input} - E_{output}$

Identify energy terms:

initial time: $E_i = KE_i + PE_i = (1/2)\, mv_i^2 + 0$

final time: $E_f = KE_f + PE_f = (1/2)\, mv_f^2 + PE_f$

transfer: $E_{output} = 0$ $E_{input} = 0$

$(1/2)mv_f^2 + PE_f - (1/2)mv_i^2 = 0$

Compare Results

One object system:

Conservation of energy

$(1/2)mv_f^2 - (1/2)mv_i^2 = 0 - (1/2)kx_f^2$

$$mv_f^2 = mv_i^2 - kx_f^2$$

$$v_f = \sqrt{v_i^2 - \frac{k}{m}x_f^2}$$

Two object system:

Conservation of energy

$(1/2)mv_f^2 + PE_f - (1/2)mv_i^2 = 0$

$$mv_f^2 = mv_i^2 - 2PE_f$$

$$v_f = \sqrt{v_i^2 - \frac{2PE_f}{m}}$$

ince the final speed does not depend on our hoice of system

$2\, PE_f = kx_f^2$

$PE_f = (1/2)\, kx_f^2$