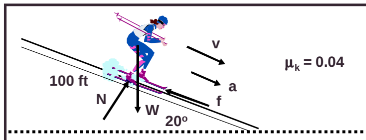
Example

While cross country skiing, you find you are on the top of a small hill. The hill side is a gentle slope at an angle of 20 degrees to the horizontal. You are new at this and are a bit afraid of going too fast since you are not good at stopping. You stop at the top of the hill and estimate that the hill side goes 100 feet before it gets to the bottom. How fast will you be going at the bottom of the hill? The coefficient of kinetic friction between your skis and the dry snow is 0.04.

We solved this problem using dynamics.

Now solve it using conservation of energy



Question:

What is the speed at the bottom of the hill?

Approach:

Use conservation of energy

Using Conservation of Energy

Conservation of Energy Equation

$E_f - E_i = E_{input} - E_{output}$

The change of energy of an system

equals

the energy transferred to (or from) that object from objects external to it.

Energy input

system energy = E_i	system	system energy = E_f
initial time		final time

Energy output

An instant	A time interval	An instant
2:36 PM	All times between 2:36 PM and 2:47 PM	2:47 PM

Choices

You choose

The system: One objects or a group of objects

Example: skier

The Initial Time

Any instant when you can determine the energy of the object.

For skier: Kinetic Energy at top

Final Time

Any later instant when you can determine the energy of the object.

For skier: Kinetic Energy at bottom

For those choices there is an in between time

Time interval between Initial Time and Final Time

Energy transfers to or from the system

Work

A force can cause a transfer of energy

3 forces on skier: Do they all cause a transfer?

Forces and Energy Transfer

Energy input

If a force transfers energy to the skier

It would cause the skier to increase her kinetic energy.

Skier speeds up

The force has a component in the direction of the skier's velocity

Energy output

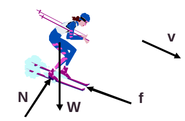
If a force transfers energy from the skier

It would cause the skier to decrease her kinetic energy.

Skier slows down

The force has a component in the opposite direction to the skier's velocity

Free-body diagram

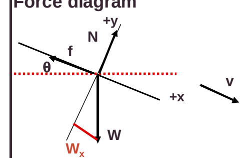


Energy inputs:

Which forces have components in direction of skier's velocity?

Cause increase in skier's kinetic energy

Force diagram



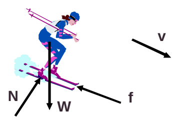
Energy input to skier caused by W_x

$$E_{input} = \int_{top}^{bottom} W_x dx$$

W_x constant:

$$E_{input} = W_x \int_{top}^{bottom} dx = W_x(x_f - x_o) = W_x L$$

Free-body diagram

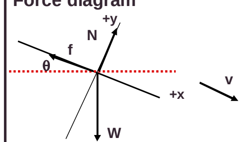


Energy outputs:

Which forces have components in opposite direction of skier's velocity?

Cause decrease in skier's kinetic energy

Force diagram



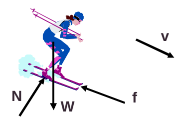
Energy output from skier caused by f

$$E_{output} = \int_{top}^{bottom} f\ dx$$

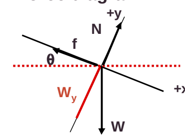
f constant:

$$E_{output} = f\int_{top}^{bottom} dx = f(x_f - x_o) = fL$$

Free-body diagram



Force diagram



Since N and W_y are perpendicular to the velocity,

They cannot transfer energy to or from the skier

Only forces with components along the velocity of an object can transfer energy to it or from it.

Relevant equations:

Conservation of Energy:

$E_f - E_i = E_{input} - E_{output}$

E_i = initial kinetic energy of skier= 0

E_i = initial kinetic energy of skier = $\frac{1}{2}mv_f^2$

E_{input} (caused by W_x) = W_xL

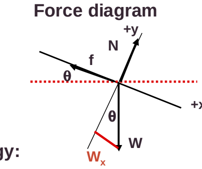
E_{output} (caused by f) = fL

W = mg

$\sin\theta = \frac{W_x}{W}$

F = μN

Force diagram



Conservation of Energy:

$$\frac{1}{2}mv_f^2 - 0 = mg\sin\theta L - \mu NL$$

Conservation of Energy:

$$\frac{1}{2}mv_f^2 - 0 = mg\sin\theta L - \mu NL$$

Solution is possible if you can get N.

Use dynamics.

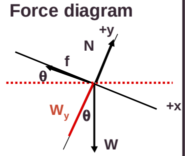
$\Sigma F_y = ma_y = 0$

$\Sigma F_y = N - W_y$

$\cos\theta = \frac{W_y}{W}$

N = mg cosθ

Force diagram



Plan:

	unknown
Find v_f	v_f
$\frac{1}{2}mv_f^2 - 0 = mg\sin\theta L - \mu NL$	N, m
Find N	
N = mg cosθ	

3 unknowns, 2 equations

Does mass cancel out?

$$\frac{1}{2}mv_f^2 - 0 = mg\sin\theta L - \mu mg\cos\theta L$$ yes

$$v_f^2 = 2gL(\sin\theta - \mu\cos\theta)$$

$$v_f = \sqrt{2gL(\sin\theta - \mu\cos\theta)}$$

Is the answer reasonable?

Evaluation

$$v_f = \sqrt{2gL(\sin\theta - \mu\cos\theta)}$$

See how v_f changes as known quantities change

As the length of the hill increases (L), v_f increases. Reasonable

If the angle of the hill to the horizontal (L), v_f were 90º,

$$v_f = \sqrt{2gL}$$

No effect of friction. Reasonable because the skier would be falling straight down, free fall.

If the angle of the hill to the horizontal (L), v_f were 0º,

$$v_f = \sqrt{-2gL\mu}$$

Not a real number so no final speed. If no slope, the skier does not move. Reasonable.

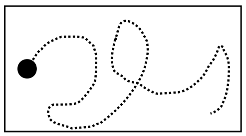
Calculating Work

A simpler example:

Block sliding on a horizontal table

Push the object around on the table.

Looking down on a table



What is the energy transferred from the block to the table?

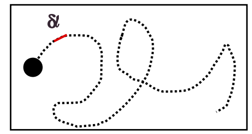
Type of interaction: friction

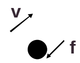
Calculate energy output for each small length of path

Add them up to get total energy output

Path

For one small length of path (δl)





Energy output (Work)

$f\,\delta l$

The force is always along the direction of travel

Add up all of the Work for the entire path

$\Sigma f(\delta\ell)$ Sum over path

If f were constant (as it is in this case)

$f\Sigma(\delta\ell) = f(L)$

More General

If f were changing along path

Add up all of the Work for the entire path

$\Sigma f(\delta\ell)$

Let δl be very small (dl) ⟶ 0

$\int f(d\ell)$ Integral over path

If f is constant (does not change over the path)

$\int f(d\ell) = f\int d\ell$

$\int (d\ell) = L$

$E_{output} = fL$

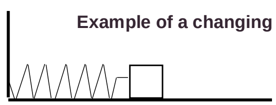
But what if the force changes along the path?

Example

You have been hired to help design a new amusement park ride for small children. In this ride, a child sits in a cart that is attached to a large spring. A mechanism give the cart a shove and the cart moves back and forth. The cart is a hovercraft that rides on a cushion of air. To check the safety of this ride, you have been asked to write a computer program that calculates the child's speed at any time as a function of the spring constant and the initial speed given by the shove. You decide to test your program in the laboratory using a block of plastic sliding on an air table. They will not let you use a real kid.

Energy Transfer by a Spring

Example of a changing force.



Block of mass m slides on a frictionless, horizontal surface. The spring is characterized by a spring constant k.

Decisions

System : block

Initial time : block at equilibrium moving left.

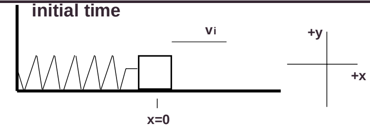
Final time : block past equilibrium moving left.

The entire motion of the block tells us there is sometimes an energy output and sometimes an energy input to the block from the spring.

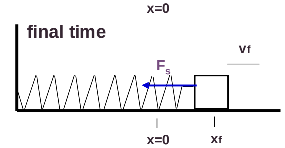
To find the energy transfer from a changing force, add up the energy transfer for each small part of the path.

Sum becomes an integral

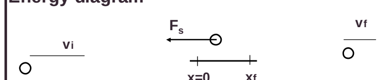
initial time



final time



Energy diagram



initial state | energy transfer | final state

$E_{initial} = \frac{1}{2}mv_i^2$

$E_{out} = \int_{x_o}^{x_f} F_x\,dx$

$F_s = k\,x$

$E_{final} = \frac{1}{2}mv_f^2$

$E_{out} = \int_{x_o}^{x_f} kx\,dx = k\int_{x_o}^{x_f} x\,dx$

$E_{out} = \frac{1}{2}kx^2\Big|_{x_o}^{x_f} = \frac{1}{2}k\left(x_f^2 - x_o^2\right)$

How to Solve Problems Using Energy

1. Picture the situation

Where is the object?
How is it moving?

Is there energy transfer?

What path does the object travel?

Carefully identify the initial time and the final time you want to consider.

Can you account for all of the energy of your object at those times?

Can you account for all energy transfers between those times?

2. Define your quantities with respect to a coordinate system.

Make sure you know which direction is + and which is -.

Force, position

3. Use your defined quantities to write down the conservation of energy equation for your situation.

Keep track of the signs.

Keep track of the target quantity.

Do you need to know anything else in addition to conservation of energy?

Force laws

Kinematics

4. Identify all unknowns in your conservation of energy equation and relate them by equations to other information or principles physics.

5. Check you answer

Correct units?

Reasonable behavior or value?