Where are we??

Textbook

By end of this week you should have read

Chap. 6

Next week finish chapter 7

Quiz – This Thursday and Friday

Textbook Chapters 1 - 6

Qualitative from Chapter 6

Problems on Dynamics

Force analysis

Acceleration from forces

Velocity & position from acceleration

At least one problem from homework – (last time all 3)

Conceptual questions you missed last time + new ones

# Interactions

We can describe the results of an interaction between two objects by using the theory of forces (Called dynamics)

Newton's 2nd Law $\sum \vec{F} = m\vec{a}$

Newton's 3rd Law

When the velocity of an object changes

There must be at least one force on that object

Caused by an interaction with another object

Use dynamics to analyze the interactions of a system by looking for change

In velocity

Now we will take a different approach to analyzing interactions by looking for what does not change.

Conservation

# Examples

Throw a ball up at speed $v_o$

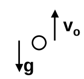
With what speed does it come down and hit your hand?

Answer: same as its initial speed.

Analyze this with the dynamics approach

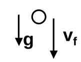
The Earth exerts a constant gravitational force on the ball in the downward direction.

This force causes the ball to have a constant downward acceleration (g)




The velocity changes

That means the ball slows down as it goes up

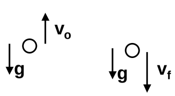



and speeds up as it comes down.

Use kinematics to calculate the final velocity of the ball

A conservation approach concentrates on

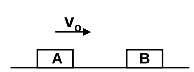
What doesn't change?



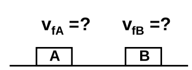

Magnitude of final velocity is the same as the magnitude of the initial velocity.

# Example

A cart moving with a constant velocity hits an equal mass cart at rest, what happens after the collision?



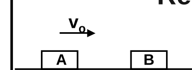

Initial time




Final time

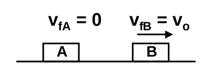
Let's try it

# Result




Initial time

$m_A = m_B$

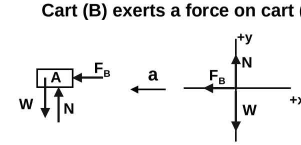



Final time

We can understand this with dynamics

During the interaction

Cart (B) exerts a force on cart (A)



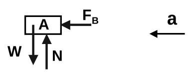

$\sum F_x = ma_x$

$F_B$ causes cart (A) to accelerate

It slows down and stops

## Collision

During the time interval of the interaction



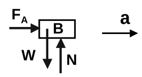

The change of cart (A)'s velocity is $v_o$

What happens to cart (B) ?

Cart (B) exerts a force on cart (A),

Cart (A) exerts and equal force on cart (B) in the opposite direction --3rd Law

$\sum F_x = ma_x$




Since the forces on cart A and B are equal

$F_A = F_B$

and their masses are equal

The accelerations of both carts are equal

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$\sum F_{Bx} = m_A a_{Ax}$ $\quad F_B = ma_A$

$\sum F_{Ax} = m_B a_{Bx}$ $\quad F_A = ma_B$

$a_A = a_B$ (not constant)

How is the final velocity of B related to the final velocity of A?

$a_A = \frac{dv_A}{dt_A}$ $\quad a_B = \frac{dv_B}{dt_B}$ $\quad \frac{dv_A}{dt_A} = \frac{dv_B}{dt_B}$

The force on A occurs over the same time interval as the force on B

$\frac{dv_A}{dt} = \frac{dv_B}{dt}$

Same acceleration over the same time interval gives the same change in velocity

Velocity of cart A went from $v_o$ to 0
Velocity of cart B goes from 0 to $v_o$
A simpler analysis using conservation.

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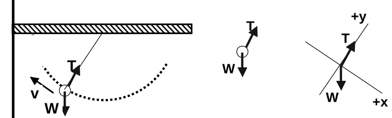
## Another System - Pendulum




Velocity changes

Motion in x direction: $\Sigma F_x = ma_x$

$W_x = ma_x$ $\quad a_x$ NOT = 0

Motion in y direction: $\Sigma F_y = ma_y$

$T - W_y = ma_y$ $\quad a_y$ NOT = 0 how do you know?

$a_x$ and $a_y$ change -- NOT constant

Trajectory is a circle - NOT constant speed

But the motion keeps repeating

Something stays the same

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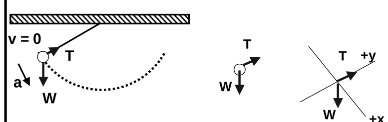
## Pendulum -- Dynamics

For example: Analyze two positions

Position 1 : Highest point in swing




$\Sigma F_x = ma_x$ $\quad W_x = ma_x$

$\Sigma F_y = ma_y$ $\quad T - W_y = ma_y$ $\quad a_y = \frac{v^2}{r}$

At this position: v = 0 so $a_y = 0$

$T = W_y$

$T < W$

but $a_x$ NOT = 0

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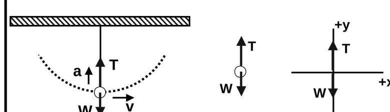
## Pendulum

Position 2 : Lowest point in swing




$\Sigma F_x = ma_x$ $\quad 0 = ma_x$

$\Sigma F_y = ma_y$ $\quad T - W = ma_y$ $\quad a_y = \frac{v^2}{r}$

At this position: $a_y$ NOT = 0

$T > W$

but $a_x = 0$

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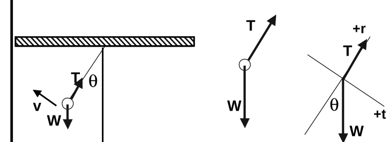
## Pendulum - Quantitative Analysis

Example of Non-constant Forces




Motion in t direction: $\Sigma F_t = ma_t$

$W_t = ma_t$

$W \sin\theta = m a_t$

$mg \sin\theta = m a_t$

$g \sin\theta = a_t$

The gravitational force, W, causes the tangential acceleration.

The tangential acceleration changes because the component of W in the tangential direction changes

$\Sigma F_t = W \sin\theta$

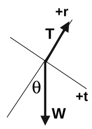
$\theta$ is a function of time

Motion in r direction: $\Sigma F_r = ma_r$

$T - W_r = ma_r$

$T - mg\cos\theta = ma_r$

$\frac{T}{m} - g\cos\theta = a_r$




From definition of acceleration and velocity for circular motion

$$a_r = \frac{v^2}{r}$$

The combination of the gravitational force, W, and the force of the string on the ball,T, cause the radial acceleration.

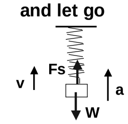
The radial acceleration changes because the component of W in the radial direction changes, and T changes

$$\Sigma F_r = T - W\cos\theta$$

$$T = m(g\cos\theta + \frac{v^2}{r})$$

## nother System - Block on a Spring

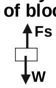
Pull block down and let go



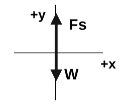

Velocity changes

Moving up, speeding up

Free-body diagram of block




Force diagram of block




Fs - W = ma

a NOT = 0

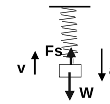
Fs > W

Velocity is changing
Acceleration is changing

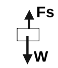
How do you know?

But the motion keeps repeating
Something stays the same

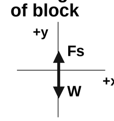
Moving up, slowing down




Free-body diagram of block




Force diagram of block




a NOT = 0

Fs - W = - ma

Fs < W

In between, there is a point where a = 0

Fs - W = 0

Fs = W

## Spring Force Behavior

imple case weight at rest hanging on a spring

If you increase the weight of the object
$\Delta y$ increases

Double the weight
doubles $\Delta y$

Theory of the spring force

$F_s = k\,\Delta y$ — $\Delta y$ measured from unstretched position

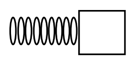
As the object gets further away from the unstretched position, the force increases.

Call position change x
$F_s = kx$

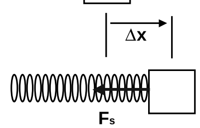
if x is measured from the unstretched position

Direction of that force is always opposite to the direction of the displacement from the unstretched position

## Spring Force

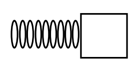


$F_s = 0$ (equilibrium position)

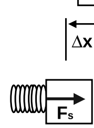



Stretched spring exerts a force on an object

Whether force is + or - depends on your coordinate system



$F_s = 0$ (equilibrium position)



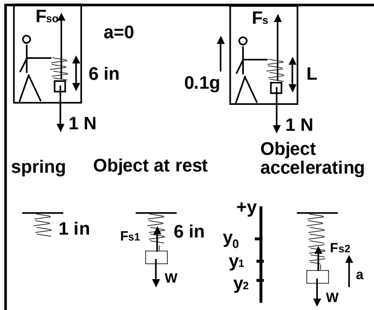

Compressed spring exerts a force on an object

Whether force is + or - depends on your coordinate system

Force is always in opposite direction from the displacement from equilibrium position

Example

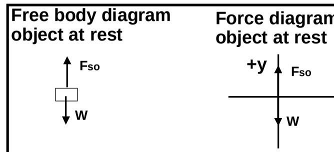
Your assignment is to make a simple, device to measure acceleration. A possible design is a spring held on one end so that it hangs vertically with a 1.0 N object on the other end. To test the device you take it to the elevators in the IDS building where, you have been told, the elevators' maximum acceleration is 0.10g. Before the elevator starts, you hang the object on the spring and it stretches from 1 inch to 6 inches. What is the length of the spring for the elevator's maximum acceleration?

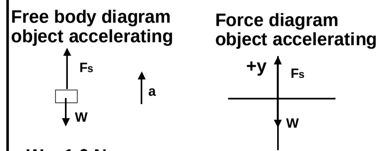



What is length of spring for accelerating object ?

Use relationship between force and accel.
Newton's 2nd law

Use force law for spring




W = 1.0 N
a = 0.1 g

$y_o = 0$
$y_1 = 6$ in

Find $y_2$

Use

$$\sum F_y = ma_y$$

$F_s = k\Delta y$

W = mg

Plan | unknowns

Find $y_2$ — $y_2$

accelerating object

$F_s = k(-y_2 + y_o)$ — $F_s$, k

$F_s = -k(y_2)$ [1]

Find $F_s$

$F_s - W = (W/g)a$ [2] — m

Find k

object at rest

$F_{so} = k(-y_1 + y_o)$

$F_{so} = -ky_1$ [3] — $F_{so}$

Find $F_{so}$

$F_{so} - W = 0$ [4]

4 unknowns, 4 equations

From 4

$F_{so} = W$ into 3

$W = -ky_1$

$$-\frac{W}{y_1} = k \quad \text{into 1}$$

$$F_s = \frac{W}{y_1} y_2 \qquad [1]$$

From 2

$$F_s - W = \frac{W}{g} a$$

$$F_s = \frac{W}{g} a + W \quad \text{into 1}$$

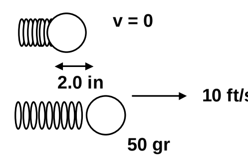
$$W\frac{y_2}{y_1} = \frac{W}{g} a + W$$

$$\boxed{y_2 = (\frac{a}{g} + 1)y_1}$$ independent of mass of object

$$1.1(6in) = y_2$$

$\boxed{y_2 = 6.6 \text{ in}}$

You are asked to choose replacement springs for a pinball machine. The spring is used to launch a small 50 gram steel ball to begin the game. In order for the game to be fun, the ball should leave the spring at a speed of 10 ft/sec. At the beginning of the game, the ball is at the end of a spring that has been compressed 2.0 inches from its unstretched length. When you release the spring, it launches the ball horizontally. Assuming that friction can be neglected, what spring constant should you choose?




Question: What is spring constant?

Approach:

Use spring force to relate the spring constant to the spring's force on the ball

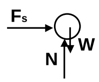
Relate that force to the acceleration of the ball using Newton's 2nd law.

Use the definition of acceleration to relate the acceleration of the ball to it's change of velocity.

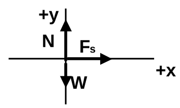
Will I need the definition of velocity also?

Neglect friction.

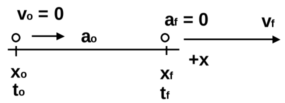
**Free body diagram of the ball**




**Force diagram of the ball**




**Motion diagram of the ball**




| | | |
|---|---|---|
| $x_o = -2.0$ in | $x_f = 0$ | m = 50 gr |
| $t_o = 0$ | $t_o = ?$ | |
| $v_o = 0$ | $v_f = 10$ ft/s | |
| $a_o = ?$ | $a_f = 0$ | |

**Target quantity: k**

**Quantitative relationships:**

$F_s = -kx$ $\quad a_x = \frac{dv_x}{dt} \quad v_x = \frac{dx}{dt}$

$\Sigma F_x = ma_x$

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**PLAN** | **unknowns**

| | | |
|---|---|---|
| **Find k** | | k |
| $F_s = -kx$ | 1 | $F_s$ |
| **Find $F_s$** | | |
| **From force diagram, forces in x direction** | | |
| $F_s = ma_x$ | 2 | $a_x$ |
| **Find $a_x$** | | |
| $a_x = \frac{dv_x}{dt}$ | 3 | t |
| **Find t** | | |
| $v_x = \frac{dx}{dt}$ | 4 | |

---

4 $\quad v_x = \frac{dx}{dt}$

$dt = \frac{dx}{v_x}$ **into** 3

$a_x = \frac{dv_x}{dt}$

$a_x = \frac{dv_x}{\frac{dx}{v_x}}$

$a_x = v_x \frac{dv_x}{dx}$ **into** 2

$F_s = mv_x \frac{dv_x}{dx}$ **into** 1

$-kx = mv_x \frac{dv_x}{dx}$

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$-kx = m\frac{1}{2}\frac{dv_x^2}{dx}$

**Find $v_x^2$ as a function of x**

**What function gives x when you take the derivative with respect to x**

$v_x^2 = bx^2 + c$

$\frac{d(v_x^2)}{dt} = 2bx$

$-kx = m\frac{1}{2}2bx$

$-\frac{k}{m} = b$

$v_x^2 = -\frac{k}{m}x^2 + c$

---

**Find c**

**at $x = x_o$ , $v_x = 0$**

$0 = -\frac{k}{m}x_0^2 + c$

$c = \frac{k}{m}x_0^2$

$v_x^2 = -\frac{k}{m}x^2 + \frac{k}{m}x_0^2$

$mv_x^2 = k(-x^2 + x_0^2)$

$\frac{mv_x^2}{(-x^2 + x_0^2)} = k$

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**Another way to solve** $\quad -kx = mv_x \frac{dv_x}{dx}$

$-kxdx = mv_x dv_x$ $\quad$ **Use integration**

$-\int kxdx = \int mv_x dv_x$

$\frac{1}{2}mv_x^2 = -\frac{1}{2}kx^2 + c$

$\frac{1}{2}mv_x^2 = -\frac{1}{2}kx^2 + c$

**Find c**

**at $x = x_o$ , $v_x = 0$**

$0 = -\frac{1}{2}kx_o^2 + c$

$c = \frac{1}{2}kx_o^2$

$\frac{1}{2}mv_x^2 = -\frac{1}{2}kx^2 + \frac{1}{2}kx_o^2$

$mv_x^2 = k(-x^2 + x_o^2)$

$\frac{mv_x^2}{-x^2 + x_o^2} = k$

Evaluate at $x_f$

$$\frac{mv_f^2}{-x_f^2 + x_o^2} = k$$

$$\boxed{\frac{mv_f^2}{x_o^2} = k}$$

$$\frac{(50 \ gr)(10 \ ft/s)^2}{(-2 \ in)^2} = k$$

$$\frac{5000 \ (gr)(ft/s)^2}{4(in)^2} = k$$

$$\frac{5000 \ (gr)(ft/s)^2 (\frac{12 \ in}{ft})^2}{4(in)^2} = k$$

$$\boxed{1.8 \times 10^5 \ gr/s^2 = k}$$

Are units of k correct?

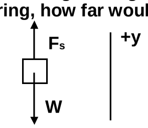
[F] = [k] [x]

[mass] [m/s ] = [k] [m]

[mass/s ] = [k] ok

Is this reasonable?

If you hung a 100 gr object on this spring, how far would it stretch




0 = ky - W

mg = ky

g (m/k) = y

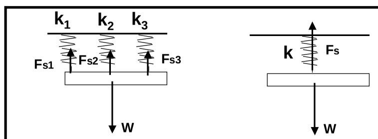
(10 m/s$^2$ )(100 gr)/(2x10$^5$ gr/s$^2$ ) = y

(1/2) x 10$^{-2}$ m = y

(1/2) cm = y a reasonable length

# Example

Your team has just completed an inexpensive prototype of a guidance system for cars. While building the prototype, your colleagues used three small springs to hold a part which is hanging vertically from the springs. The three springs have the same length and each has one end attached to a rigid bar and the other end attached to the part. Precise adjustments have been made to the motion of the part by using a different spring constant for each spring. These spring constants are given in the design report. To make the final design less expensive and more reliable, your manager tells you to replace the three springs with a single spring, with the specifications you are to determine, without changing the design of the system.




Find the spring constant of a single spring which has the same behavior as the 3 springs

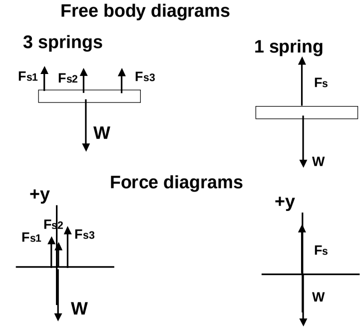
Use spring force law

Single spring should have the same displacement and acceleration as the 3 springs when the object is hanging on it

Use relationship between force and accel

Use Newton's 2nd law

Free body diagrams




Given : $k_1$, $k_2$, $k_3$ , W

Target quantity: k

Use

$\sum F_y = ma_y$ $F_s = ky$

| | | unknowns |
|---|---|---|
| Find k | | k |
| single spring | | |
| $F_s = ky$ | 1 | $F_s$ , y |
| Find $F_s$ | | |
| $\sum F_y = F_s - W$ | 2 | $\Sigma F_Y$ |
| Find $\Sigma F_Y$ | | |
| $\sum F_y = ma_y$ | 3 | m, $a_y$ |
| Find m | | |
| W = mg | 4 | |
| Find $a_y$ | | |
| three springs | | |
| $\sum F'_y = ma_y$ | 5 | $\Sigma F'_Y$ |
| Find $\Sigma F'_Y$ | | |
| $\sum F'_y = F_{s1} + F_{s2} + F_{s3} - W$ | 6 | $F_{s1}$ , $F_{s2}$ , $F_{s3}$ |
| Find $F_{s1}$ , $F_{s2}$ , $F_{s3}$ | | |
| $F_{s1} = k_1y$ | 7 | |
| $F_{s2} = k_2y$ | 8 | |
| $F_{s3} = k_3y$ | 9 | |

Is the problem solved?

10 unknowns, 9 equations

Which unknown is missing?

Do we know anything else useful?

Can't think of anything.

Try for a solution anyway

Will any unknowns (especially y) cancel ?

Check plan

Put 7, 8, 9 into 6

$\sum F'_y = k_1 y + k_2 y + k_3 y - W$ into 5

$k_1 y + k_2 y + k_3 y - W = ma_y$ into 3

$\sum F_y = k_1 y + k_2 y + k_3 y - W$ into 2

$F_s - W = k_1 y + k_2 y + k_3 y - W$

$F_s = k_1 y + k_2 y + k_3 y$ into 1

$ky = k_1 y + k_2 y + k_3 y$ yes! y cancels out

$\boxed{k = k_1 + k_2 + k_3}$

Another plan based on another approach

The only influence on the motion of the object

Forces exerted by other objects

Earth, Springs

If the sum of the forces on the object is the same for 3 springs and 1 spring the motion will be the same

| Plan | | unknowns |
|---|---|---|
| Find k single spring | | k |
| $F_s = ky$ | 1 | $F_s$ , y |
| Find $F_s$ $\sum F_y = F_s - W$ | 2 | $\Sigma F_Y$ |
| Find $\Sigma F_Y$ $\sum F_y = F_{s1} + F_{s2} + F_{s3} - W$ | 3 | $F_{s1}$ , $F_{s2}$ , $F_{s3}$ |
| Find $F_{s1}$ , $F_{s2}$ , $F_{s3}$ | | |
| $F_{s1} = k_1 y$ | 4 | |
| $F_{s2} = k_2 y$ | 5 | |
| $F_{s3} = k_3 y$ | 6 | |

Put 4, 5, 6 into 3

$\sum F_y = k_1 y + k_2 y + k_3 y - W$ into 2

$F_s - W = k_1 y + k_2 y + k_3 y - W$

$F_s = k_1 y + k_2 y + k_3 y$ into 1

$ky = k_1 y + k_2 y + k_3 y$ y cancels out

$\boxed{k = k_1 + k_2 + k_3}$

## Conservation

What do we mean by saying a quantity is conserved?

Experience:

Conservation of money.

System: bank account

| Bank account $A | Bank account $A |
|---|---|
| initial time | final time |

$\Delta M_{system} = M_f - M_i$

$M_i = M_f$

$\Delta M_{system} = 0$

ok unless something happens

Withdrawals

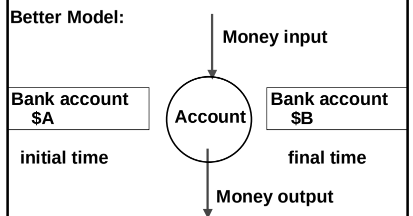
Deposits

Interest

Bank Charges

## Conservation of money

Better Model:




$M_{initial} + M_{input} - M_{output} = M_{final}$

$M_{input} - M_{output} = M_{final} - M_{initial}$

$\Delta M_{transfer} = \Delta M_{system}$

Money is conserved for your bank account

If no connection to outside world

Isolated system

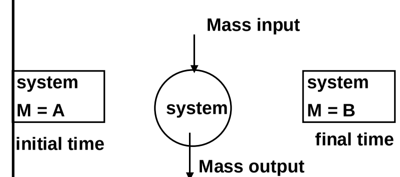
$M_{input} = 0$

$M_{outpur} = 0$

$\Delta M_{transfeputr} = 0$

$\Delta M_{system} = 0$

## CONSERVATION OF MASS




$M_i + M_{in} - M_{out} = M_f$

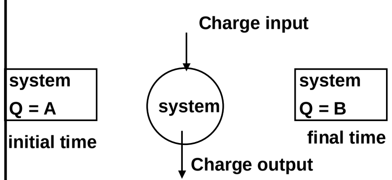
$M_{in} - M_{out} = M_f - M_i$

$\Delta M_{transfer} = \Delta M_{system}$

If 16 grams of oxygen combines completely with 2 grams of hydrogen to form water, how many grams of water are made?

If 16 grams of oxygen are combined with 4 grams of hydrogen to make 18 grams of water with some left over hydrogen, how much hydrogen is left over?

# CONSERVATION OF CHARGE




$Q_i + Q_{in} - Q_{out} = Q_f$

$Q_{in} - Q_{out} = Q_f - Q_i$

$\Delta Q_{transfer} = \Delta Q_{system}$

If a capacitor is charged to 16 microcoulombs and then connected to another uncharged capacitor. 10 minutes later the charge on the first capacitor is measured to be 5 microcoulombs. What is the charge on the second capacitor?

# Motion and Interactions

What is conserved?

Consider an isolated system (object)

Object velocity doesn't change

Object mass doesn't change

Good candidates

What characterizes an interaction ?

Larger mass NEEDS "Stronger" interaction to have an affect

Larger speed NEEDS "Stronger" interaction to have an affect

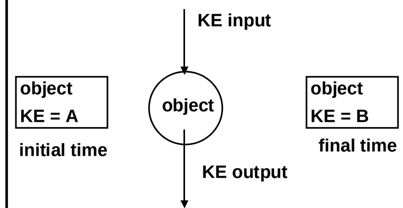
Try to invent a quantity to be conserved that is a combination of mass and speed

Motion Energy (Kinetic Energy)

$KE = (1/2)\, m\, v^2$

Is it conserved?

# Kinetic Energy




Is it true that?

$KE_i + KE_{in} - KE_{out} = KE_f$

$\Delta KE_{transfer} = \Delta KE_{system}$

Example, drop some object

$KE_i = 0$

$KE_f$ NOT = 0

No moving things interacting with object

$\Delta KE_{transfer} = 0$

Kinetic Energy is <u>NOT</u> conserved

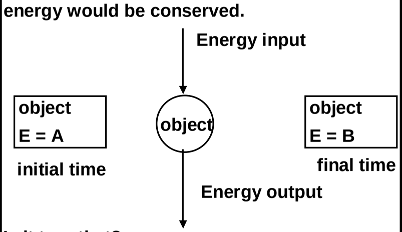
# Energy

We would like something related to kinetic energy to be conserved

New theory

Call the conserved quantity energy

Kinetic energy is only one type of the energy. If we could keep track of all types of energy (like types of money in your bank account) , energy would be conserved.




Is it true that?

$E_i + E_{in} - E_{out} = E_f$

$\Delta E_{transfer} = \Delta E_{system}$

What is $E_{in}$ or $E_{out}$?

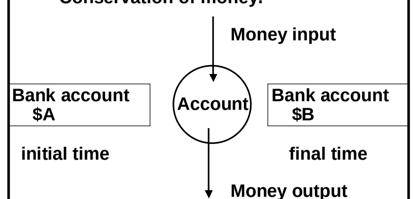
# Identifying Energy

Kinetic Energy is one example of energy

How to find the other forms.

Bank account example

Conservation of money.




$\Delta M_{transfer} = \Delta M_{system}$

If you can't balance your checking account

Check your math

Look for new forms of money

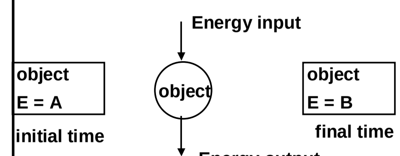
Bank charges for check writing

# Example

Drop an object

Initial time: just after release

Final time: just before it hits




$E_i + E_{in} - E_{out} = E_f$

$E_{in} - E_{out} = E_f - E_i$

$\Delta E_{transfer} = \Delta E_{system}$

$E_i = KE_i = (1/2)\, m\, v_i^2 = 0$

$E_f = KE_f = (1/2)\, m\, v_f^2$

$E_{in} = ?$

$E_{out} = ?$

$E_{out} = 0$

## Energy Transfer

Energy transfer to an object is caused by
Interactions with other objects

$E_{in}$ related to force on object

Dimensional analysis to give hint

(1/2) m $v^2$ = (something) F

[kg] $[m/s]^2$ = [something] [N]

[kg] $[m/s]^2$ = [something] [kg] $[m/s^2]$

[m] = [something]

something is a position, or displacement, or distance

---

(1/2) m $v^2$ = (something) F

Construct the theory, for dropping an object.

The velocity just before hitting the ground increases if the height increases

Vertical displacement matters

(1/2) m $v^2$ = F (vertical displacement)

$E_{in} = F\,\Delta y$ where y is the vertical direction

F is in same direction as $\Delta y$

Object speeds up

Object gains energy

The energy is input

Is theory correct?

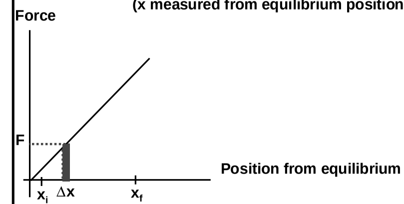
---

What if the force on an object is not constant during the motion?

Example: Spring

$\vec{F} = -k\vec{x}$

(x measured from equilibrium position)




For a small displacement

Force does not change much

$E_{input} = F\,\Delta x$ Area of small rectangle

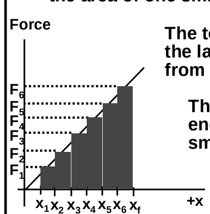
Total displacement from $x_i$ to $x_f$ =

Sum of small displacements

---

## Adding Small Rectangles

The energy input for a small displacement is $F\Delta x$
the area of one small rectangle




The total energy input for the large displacement from $x_1$ to $x_f$

The sum of all the small energy inputs from the small displacements

area under the line

$$E_{input} = \lim_{\Delta x \to 0} \sum F\,\Delta x$$

Calculus is very useful to get the area

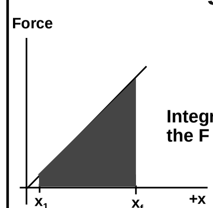
$$\lim_{\Delta x \to 0} \sum F\Delta x = \int F dx$$

Integral is the area under the F vs x curve.

$$E_{input} = \int F dx$$

---

## Integral




$$E_{input} = \int F dx$$

Integral is the area under the F vs x curve.

Red area is the difference of the area of the two triangles

$$A = \frac{1}{2}(F_f x_f) - \frac{1}{2}(F_1 x_1)$$

$$A = \frac{1}{2}((kx_f)x_f) - \frac{1}{2}((kx_1)x_1)$$

$$A = \frac{1}{2}(kx_f^2) - \frac{1}{2}(kx_1^2)$$

$$A = \frac{1}{2}k(x_f^2 - x_1^2) = E_{input}$$

---

## Integral

In calculus the integral is a sum

$$\lim_{\Delta x \to 0} \sum F\Delta x = \int F dx$$

Also the integral is the anti-derivative

$\int F dx$ Means what function gives a derivative with respect to x that equals F?

For a spring: F = kx

What function gives kx as its x derivative?

$$\frac{1}{2}kx^2$$

$$\int F dx = \int kx dx = \frac{1}{2}kx^2$$

Determine the integral for our displacement

From $x_1$ to $x_f$

$$\int_{x_1}^{x_f} F dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_1^2 = E_{input}$$

## Object goes up

Throw an object straight up

It slows down

Must be an energy output

$$E_{output} = \int F dx$$

If there is energy input or output

Energy is transferred

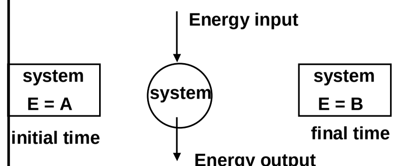
$$E_{transfer} = \int F dx$$

$$\Delta E_{transfer} = E_{input} - E_{output}$$

Conservation of energy

$$E_f - E_i = \Delta E_{transfer}$$

## Conservation of Energy




$E_{in} - E_{out} = E_f - E_i$

$\Delta E_{system} = \Delta E_{transfer}$

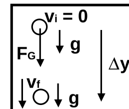
If your system is a single object
the energy of the system can ONLY be

KINETIC ENERGY $\frac{1}{2}mv^2$

Energy transfer due to a force is $\int F_r dr$
Called WORK

For complex systems (more than 1 object)
there are other types of energy

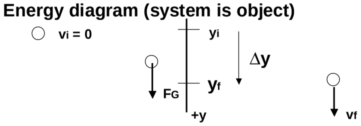
## Example




Drop an object. What is its speed after it falls a distance $\Delta y$ ?

Approach: Use conservation of energy.
Choose system to be the object
External force on the object is the gravitational force (weight)
Input energy
Final speed does not depend on mass of object
For a falling object (y vertical, + down)

Energy diagram (system is object)




target quantity: $v_f$ (as a function of $\Delta y$)

---

Relevant equations:

$$E_f - E_i = E_{in} - E_{out}$$

$$E_{in} = \int_{y_i}^{y_f} F_G dy$$

$E_{out} = 0$

$E_i$ = KE = (1/2)$mv_i^2$ = 0

$E_f$ = KE = (1/2)$mv_f^2$

$F_G$ = mg (near the surface of the Earth)

Plan | unknowns
Find $v_f$ | $v_f$
conservation of energy

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \int_{y_i}^{y_f} F_G dy = \int_{y_i}^{y_f} mgdy \qquad m$$

$$\int_{y_i}^{y_f} F_G dy = \int_{y_i}^{y_f} mgdy = mg\int_{y_i}^{y_f} dy = mg(y_f - y_i)$$

2 unknowns and only 1 equation

all terms in the energy equation depend on m

m should cancel out of problem

---

$$\frac{1}{2}\cancel{m}v_f^2 - \frac{1}{2}\cancel{m}v_i^2 = \cancel{m}g(y_f - y_i)$$

m cancels
as expected

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_i^2 = g(y_f - y_i)$$

$$v_f^2 - v_i^2 = 2g(y_f - y_i)$$

$$v_f^2 = 2g(y_f - y_i)$$

Is this correct?

Check by doing the problem the old way

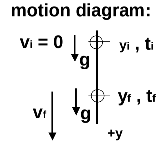
---

approach: use kinematics

Acceleration a = g = constant

Initial velocity $v_i$ = 0

motion diagram:




target quantity : $v_f$

Relevant equations:

$$a = \frac{\Delta v}{\Delta t} = \frac{(v_f - v_i)}{\Delta t}$$

since a is a constant

$$y_f - y_i = \frac{1}{2}a(\Delta t)^2 + v_0\Delta t$$

**Plan:**

**unknowns**

**Find $v_f$** $\qquad$ $v_f$

$$g = \frac{v_f - v_i}{\Delta t}$$

$$g = \frac{v_f}{\Delta t} \qquad [1] \qquad \Delta t$$

**Find $\Delta t$**

$$y_f - y_i = \frac{1}{2} g (\Delta t)^2 + v_0 \Delta t$$

$$y_f - y_i = \frac{1}{2} g (\Delta t)^2 \qquad [2]$$

**2 unknowns, 2 equations** $\qquad$ **ok**

**solve 1 for $\Delta t$ and put into 2**

$$\Delta t = \frac{v_f}{g}$$

$$y_f - y_i = \frac{1}{2} g \left( \frac{v_f}{g} \right)^2$$

$$2g(y_f - y_i) = v_f^2$$

**yes both approaches give same answer**