This week
Applications of Forces and Torques
Chap. 12, sec. 1-5
Conservation of angular momentum
Chap. 10, sec. 1-4

Last week
Oscillations
Chap. 14

Final Exam
covering the entire semester
Extra time granted about 1 hour
about 5 Problems
about 30 Multiple Choice
Early start for those who want it - 6 pm
Each problem will typically involve several
fundamental physics concepts.

Example
You have a part time job with a company
that designs loading equipment for
freight. Your team has designed a simple
crane for lifting heavy cargo. A 45.0 kg
bar 15 ft long is made out of lightweight
aluminum and is supported at its base by a
hinge that allows the bar to pivot vertically.
A support cable runs from the other end of
the bar that is in the air to the ground.
The team is worried that the hinge might fail.
Your task is to determine the force on the
hinge when a package of 225 kg is lifted
straight up into the air from the end of the
bar. You have been asked to consider the
case where the bar is at an angle of 45° to
the ground and the support cable is at an
angle of 30° to the ground. The package is
lifted with an acceleration of 0.10g.

Starting a Solution
Three possible approaches to solving
physics problems

Kinematics - Linear and rotational
• Position
• Time
• Time
• Velocity
• Angular velocity
• Angular acceleration

Dynamics - Linear and rotational
Add Forces
Add Torques
Newton’s 2nd Law
Newton’s 3rd Law
Conservation - Linear and rotational

Energy
Momentum
Angular Momentum

Forces
Free body diagram of bar
\[ \sum F_x = H_x - C_x \cdot W_b - T = 0 \]
\[ \sum F_y = H_y - C_y \cdot W_b = 0 \]
Free body diagram of package
\[ H_x^2 + H_y^2 = H^2 \]
\[ C_x = C \sin \theta \]
\[ C_y = C \cos \theta \]
\[ \sum F_y = T - W_p \]
\[ W = mg \]
Review

Defined the bar as the system used
\[ \sum F = m a \]
\[ \sum F_x = 0 \]
\[ \sum F_y = 0 \]
needed to find angles

\[ \sum \tau = 0 \]
\[ \tau = r F \]
\[ r = r F \]
took hinge as axis of rotation

Defined package as the system
\[ \sum F = m a \]
on package to get T

Organize the algebra

Rotations and Conservation

An ice skater is spinning in place on the ice when he brings his arms in close to his body. Determine his angular speed as a function of his initial and final moment of inertia and initial angular speed.

\[ I_f = \frac{1}{2} m_r \]
\[ I_i = \frac{1}{2} m_r \]

Possible approaches

Dynamics (vector)

Conservation of Energy (scalar) \[ KE = \frac{1}{2} I \omega^2 \]

Use energy ---- system: skater

\[ \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2 = 0 \]
\[ \omega_f^2 = \frac{I_i}{I_f} \omega_i^2 \]
\[ \omega_f = \frac{I_i}{I_f} \omega_i \]

If \( I_i < I_f \) Is it true?
If $I$ increases by a factor of 4, $\omega$ decreases by a factor of 2.

Do an experiment like this in lab.

Increased $I$ system: disk, ring

This prediction does not work out. Determine actual relationship in lab.

Need an external interaction: external torque

$\tau = F \times r$

$\frac{d\omega}{dt} = \frac{\Delta I}{I}$

$\int \frac{d\omega}{dt} dt = \int \frac{d\Delta I}{I} dt$

$\int \frac{d\Delta I}{I} dt = I(\omega_f - \omega_i)$

$\int \frac{d\tau}{dt} dt = L_f - L_i$

$\int \frac{d\omega}{dt} dt = \Delta I_{\text{transfer}}$

Angular momentum is conserved

$\Delta L_{\text{system}} = \Delta L_{\text{transfer}}$

$L_f - L_i = \Delta L_{\text{transfer}}$

$\frac{dL}{dt} = \tau$

A 250 g hockey puck traveling across the ice at 5.0 ft/sec hits the end of a 1.0 kg hockey stick that is laying at rest on the ice. The puck hits the hockey stick 3.0 ft from its center of mass. The puck bounces straight back at 1.0 ft/sec. How does the hockey stick move just after it is hit? The moment of inertia of the hockey stick rotating about its center of mass is 0.10 kg m$^2$.
What is the velocity of the center of mass of the hockey stick and the angular velocity about the center of mass?

Use conservation of momentum. Conservation of energy is useless. Why?

Use conservation of angular momentum. Use conservation of angular momentum system: puck + stick initial time: just before the collision final time: just after the collision

Does the hockey puck have a moment of inertia? Depends on the axis of rotation

Angular momentum of hockey puck just before the collision

Angular momentum of stick just before collision?

What is the angular momentum of system just before the collision?

Does the system have angular momentum after the collision?

Is there any angular momentum transfer to the system?

Does the system have angular momentum before the collision?

Gives the velocity of center of mass! Needs to get the angular velocity.

Conservation of momentum: no momentum transfer

\[(m_p v_f - m_p v_0) - m_p v_0 = 0\]

Gives the velocity of center of mass! Need to get the angular velocity.

Angular momentum: puck + stick

conservation of angular momentum: \(I_f - I_b = 0\) No external torque

Does the system have angular momentum after the collision?

Conservation of angular momentum:

\[L_f - L_b = 0\]

Direction out Choose out +

Angular momentum of hockey puck just before the collision

\[L = (m_p r^2) \frac{v_p}{r} = m_p v_0 \]

Direction out

Angular momentum of stick just before collision?

What is the angular momentum of system just before the collision?

Check units

Note that there was no need to convert to consistent units.
Conservation of Angular Momentum

\[ -m_p r^2 \frac{v_f}{r} + l_0 \omega - m_p r^2 \frac{v_o}{r} = 0 \]

Find \( \omega \)

\[ I_0 \omega = m_p \frac{r^2 v_f}{r} = m_p \frac{r^2 v_t}{r} \]
\[ I_0 \omega = m_p (v_x + v_t) \]
\[ \omega = \frac{m_p (v_x + v_t)}{I_0} \]

check units
put in numbers

Just before the puck hits the stick it has an angular momentum with respect to the center of mass of the stick.

\[ L = I_0 \omega \]
\[ L = (m_p r^2) \frac{v_f}{r} = m_p rv_o \]

What is the angular momentum of the puck with respect to the center of mass of the stick some time before that?

\[ \omega = \frac{v_t}{r} \]
\[ v_t \] is the component of \( v \) perpendicular to \( r \)

Does the angular momentum of the puck change as it moves toward the stick?

\[ \omega = \frac{v_t}{r} \]

Angular momentum of puck

\[ L = I_0 = (m_p r^2) \frac{v_t}{r} = m_p rv_t \]
\[ L = m_p r v_t = mv d \] Does not change!!!

Angular Momentum as a Cross Product

\[ \vec{L} = \vec{r} \times \vec{p} \]

Another expression for angular momentum of a particle

Magnitude of angular momentum \( r \) \( \rho \)

direction \( r(\rho \cos\phi) \)
right hand rule: from \( r \) to \( p \)
out

Conservation of angular momentum

\[ \Delta \vec{L}_{\text{system}} = \Delta \vec{L}_{\text{transfer}} \]

Any change in the angular momentum of a system must come from

Interactions with objects outside the system

You choose the system

System bicycle wheel

Direction of Angular Momentum of system

Initial: down Final: up
\[ \phi : \text{down} \quad \phi : \text{up} \]
turn it over

Another system

System bicycle wheel + person + stool
Chair is free to turn (no external torque)

No Angular Momentum transfer

Initial
\[ \phi : \text{down} \]
\[ \phi : \text{up} \]

Final
\[ \phi : \text{down} \]

Direction of Angular Momentum of system

Initial: down Final: down

Angular Momentum transfer by ?
Angular Momentum Transfer

Balanced
pivot point

Now apply a force

The force causes a torque on the system
\( \tau = \vec{r} \times \vec{F} \)
Torque direction: out
The torque causes angular momentum transfer to the system
\( \int \tau \, dt = \Delta L_{\text{transfer}} \)
Angular momentum transfer direction: out

\( \Delta L_{\text{transfer}} \) is out

Conservation of angular momentum
\( L_f - L_i = \Delta L_{\text{transfer}} \)
\( L_f = \Delta L_{\text{transfer}} \) is out
\( L_i = 0 \) is out
System turns up

What happens if wheel is spinning?
The rotating wheel has a large angular momentum

How does the system move?

When the bar turns
angular momentum transfer is still perpendicular to the bar
Bar turns again so that wheel angular momentum in direction of final angular momentum

The bar keeps turning
\( \omega_{\text{bar}} \) is up

Angular momentum from the wheel is always along the bar
Bar turns so that wheel angular momentum in direction of final angular momentum