This week

Applications of Forces and Torques

Chap. 12, sec. 1- 5

Conservation of angular momentum

Chap. 10, sec. 1- 4

Last weeks

Oscillations

Chap. 14

Final Exam covering the entire semester

Extra time granted about 1 hour

about 5 Problems

about 30 Multiple Choice

Early start for those who want it - 6 pm

Each problem will typically involve several fundamental physics concepts.

Starting a Solution

Three possible approaches to solving physics problems

Kinematics - Linear and rotational

• Position	• Angle
• Time	• Time
• Velocity	• Angular Velocity
• Acceleration	• Angular Acceleration

What the quantities mean.

What the quantities are equal to.

Dynamics - Linear and rotational

Add Forces Add Torques

Newton's 2nd Law

Newton's 3rd Law

Conservation - Linear and rotational

$X_f - X_i = X_{transfer}$

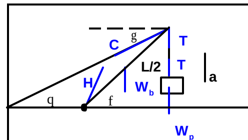
Energy

Momentum

Angular Momentum

Example

You have a part time job with a company that designs loading equipment for freight. Your team has designed a simple crane for lifting heavy cargo. A 45.0 kg bar 15 ft long is made out of lightweight aluminum and is supported at its base by a hinge that allows the bar to pivot vertically. A support cable runs from the other end of the bar that is in the air to the ground. The team is worried that the hinge might fail. Your task is to determine the force on the hinge when a package of 225 kg is lifted straight up into the air from the end of the bar. You have been asked to consider the case where the bar is at an angle of 45° to the ground and the support cable is at an angle of 30° to the ground. The package is lifted with an acceleration of 0.10g.



$\theta = 30°$

$\phi = 45°$

$W_p = (225\text{ kg})g$

$W_b = (45\text{ kg})g$

$a = 0.10\text{ g}$

$L = 15\text{ ft}$

$H = ?$

What is the force on the hinge?

Use dynamics on the bar

Sum horizontal forces = $ma_h = 0$

Sum vertical forces = $ma_v = 0$

Sum torques out of plane = $I\alpha = 0$

Take axis of rotation as hinge

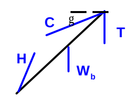
Geometry of angles to get γ

$\gamma = \theta = 30°$

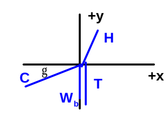
$T > W_p$ because package is accelerating up

Forces

Free body diagram of bar



Force diagram of bar



$\Sigma F_y = H_y - C_y - W_b - T = 0$

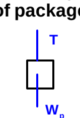
$\Sigma F_x = H_x - C_x = 0$

$H_x^2 + H_y^2 = H^2$

$C_y = C \sin\gamma$

$C_x = C \cos\gamma$

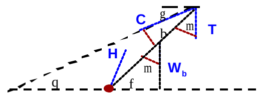
Free body diagram of package



$\Sigma F_y = T - W_p$

$W = mg$

Torque



St = L C_t - L T_t - (L/2) W_{bt} = 0

C_t = C sinb

T_t = T sinm

W_{bt} = W_b sinm

Geometry

g + b = f

b = 45° - 30° = 15°

m + f + 90° = 180°

m = 90° - 45° = 45°

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unknowns

Find H — H

$H_x^2 + H_y^2 = H^2$ [1] — H_x, H_y

Find H_x (Forces)

H_x - C cosg = 0 [2] — C

Find C (Torque)

L Csinb - L Tsinm - (L/2) m_bg sinm = 0

Csinb - Tsinm - (1/2) m_bg sinm = 0 [3] — T

Find T (Forces on package)

T - m_pg = m_p a [4]

Can solve for H_x

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[4] T - m_pg = m_p a Solve for T put in [3]

T = m_p (a + g)

Csinb - m_p (a + g) sinm - (1/2) m_bg sinm = 0

Solve for C put into [2]

Csinb = m_p (a + g) sinm + (1/2) m_bg sinm

C = (m_p (a + g) sinm + (1/2) m_bg sinm) / sinb

H_x = cosg (m_p (a + g) sinm + (1/2) m_bg sinm) / sinb

Check units

Ok units of force = units of ma

Everything is known

Just plug in the numbers

H_x = cos30° ((225kg) (9.8) (m/s²)(1.1) sin45° + (1/2)(45kg) (9.8) (m/s²) sin45°) / sin15°)

Find H_y (Forces)

H_y - C sing - m_bg - T = 0

Find C (from before)

C = (m_p (a + g) sinm + (1/2) m_bg sinm) / sinb

Find T (from before)

T = m_p a + m_p g

H_y = sing ((m_p (a + g) sinm + (1/2)m_bg sinm) / sinb) + m_bg + m_p (a + g)

Check units

Everything is known

just plug in the numbers

Finally

$H_x^2 + H_y^2 = H^2$

Review

Defined the bar as the system

used

$\text{å}\vec{F} = m\vec{a}$

å F_x = 0

å F_y = 0 — needed to find angles

å $\vec{t}$ = 0 — took hinge as axis of rotation

$\vec{t} = \vec{r}$ ´ $\vec{F}$

t = rF_t — needed to find angles

Defined package as the system

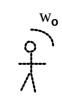
$\text{å}\vec{F} = m\vec{a}$ on package to get T

Organize the algebra

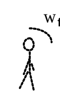
Rotations and Conservation

An ice skater is spinning in place on the ice when he brings his arms in close to his body. Determine his angular speed as a function of his initial and final moment of inertia and initial angular speed.

initial I_o



final I_f



Possible approaches

Dynamics (vector) å t = I a

Conservation of Energy (scalar) KE = $\frac{1}{2}$Iw²

Use energy ---- system: skater

$\frac{1}{2} I_f w_f^2 - \frac{1}{2} I_o w_o^2 = 0$

$w_f^2 = \frac{I_o}{I_f} w_o^2$

$w_f = \sqrt{\frac{I_o}{I_f}} w_o$ — if $I_f < I_o$ — $w_f > w_o$

Is it true?

Prediction and Experiment

check quantitatively with experiment

$$\omega_f = \sqrt{\frac{I_o}{I_f}}\,\omega_o$$

prediction

if I increases by a factor of 4

ω decreases by a factor of 2

Do an experiment like this in lab

Increased I

system: disk, ring

This prediction does not work out

Determine actual relationship in lab

What is wrong

Conservation of Energy

forgot the change of internal energy

$$\frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_o\omega_o^2 + \Delta E_{internal} = 0$$

In case of skater

Internal energy decreases to pull arms in

In case of disk and ring

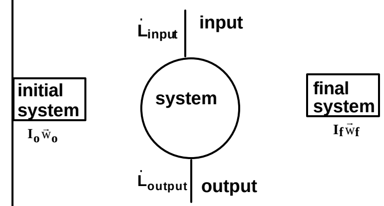
Internal energy increases in inelastic collision

Don't know the change of internal energy

Conservation of energy is not useful.

Conservation of angular momentum

$\vec{L} = I\vec{\omega}$



$$\vec{L}_f - \vec{L}_i = \vec{L}_{input} - \vec{L}_{output}$$

$$\Delta\vec{L}_{transfer} = \vec{L}_{input} - \vec{L}_{output}$$

In case of skater or ring dropped on disk

No external interaction

No angular momentum transfer

$$I_f\omega_f - I_o\omega_o = 0$$

$$\omega_f = \frac{I_o}{I_f}\omega_o$$

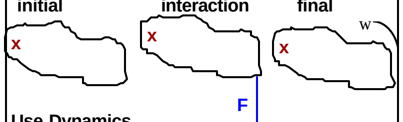
If I increases by a factor of 4

ω decreases by a factor of 4

Angular momentum transfer

Need an external interaction: external torque

initial interaction final



Use Dynamics

$$L_f - L_i = \Delta L_{transfer}$$

$$\tau = I\alpha$$

$$\tau = I\frac{d\omega}{dt}$$

$$\int \tau\, dt = \int I\frac{d\omega}{dt}dt = I\int \frac{d\omega}{dt}dt$$

$$\int_{interaction\ time} \tau\, dt = I(\omega_f - \omega_i)$$

$$\int_{interaction\ time} \tau\, dt = L_f - L_i$$

$$\int_{interaction\ time} \tau\, dt = \Delta L_{transfer}$$

Angular momentum

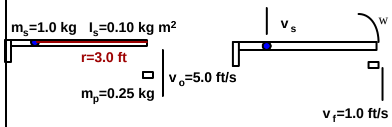
Angular momentum is conserved

$$\Delta\vec{L}_{system} = \Delta\vec{L}_{transfer}$$

$$\vec{L} = I\vec{\omega}$$

$$\Delta L_{transfer} = \int_{interaction\ time} \tau\, dt$$

A 250 g hockey puck traveling across the ice at 5.0 ft/sec hits the end of a 1.0 kg hockey stick that is laying at rest on the ice. The puck hits the hockey stick 3.0 ft from its center of mass. The puck bounces straight back at 1.0 ft/sec. How does the hockey stick move just after it is hit? The moment of inertia of the hockey stick rotating about its center of mass is 0.10 kg m^2.



What is the velocity of the center of mass of the hockey stick and the angular velocity about the center of mass?

Use conservation

Conservation of energy is useless. Why?

Use conservation of momentum

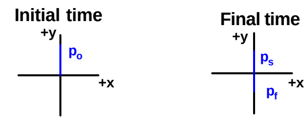
Use conservation of angular momentum

system: puck + stick

initial time: just before the collision

final time: just after the collision

Momentum: system: puck + stick



conservation of momentum: no momentum transfer $(m_s v_s - m_p v_f) - m_p v_o = 0$

Gives the velocity of center of mass!

Need to get the angular velocity.

Angular momentum: system: puck + stick

Is there any angular momentum transfer to the system?

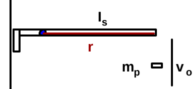
conservation of angular momentum:

$\vec{L}_f - \vec{L}_i = 0$ No external torque

Does the system have angular momentum after the collision?

Does the system have angular momentum before the collision?

Before Collision



What is ω for the system ?

$$\omega = \frac{v_t}{r}$$

just before collision $\omega_o = \frac{v_o}{r}$

An object moving in a straight line can have an angular velocity

Depends on axis of rotation

Initial angular momentum

$L_i = I_p \omega_o = I_p \frac{v_o}{r}$ Direction: same as ω out

Does the hockey puck have a moment of inertia?

Depends on the axis of rotation

$$I = m_p r^2$$

Angular momentum of hockey puck just before the collision

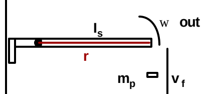
$L = (m_p r^2)\frac{v_o}{r} = m_p r v_o$ Direction out

Choose out as +

Angular momentum of stick just before collision?

What is the angular momentum of system just before the collision?

After Collision



$\vec{L}_{system} = \vec{L}_{puck} + \vec{L}_{stick}$ Direction?

Conservation of angular momentum

$$\vec{L}_f - \vec{L}_i = 0$$

Final direction same as initial direction: out

$$L_{z\ final} - L_{z\ initial} = 0$$

$$-m_p r^2 \frac{v_f}{r} + I_s \omega - m_p r^2 \frac{v_o}{r} = 0$$

Gives angular velocity about center of mass

Solves the problem:

Objects moving in straight lines can have angular momentum !!!

Conservation of Momentum

$$(m_s v_s - m_p v_f) - m_p v_o = 0$$

Find v_s

$$m_s v_s = m_p v_o + m_p v_f$$

$$v_s = \frac{m_p}{m_s}(v_o + v_f)$$

check units

$$v_s = \frac{0.25\text{kg}}{1.0\text{kg}}\left(5.0\frac{ft}{s} + 1.0\frac{ft}{s}\right)$$

$$v_s = 1.50\frac{ft}{s}$$

note that there was no need to convert to consistent units.

Conservation of Angular Momentum

$$-m_p r^2 \frac{v_f}{r} + I_s\omega - m_p r^2\frac{v_o}{r} = 0$$

Find ω

$$I_s\omega = m_p r^2\frac{v_o}{r} + m_p r^2\frac{v_f}{r}$$

$$I_s\omega = m_p r(v_o + v_f)$$

$$\omega = \frac{m_p r(v_o + v_f)}{I_s}$$

check units

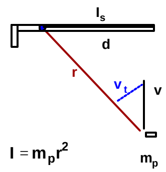
put in numbers

Just before the puck hits the stick it has an angular momentum with respect to the center of mass of the stick.

$$\vec{L} = I\vec{\omega}$$

$$L = \left(m_p r^2\right)\frac{v_o}{r} = m_p r v_o$$

What is the angular momentum of the puck with respect to the center of mass of the stick some time before that?

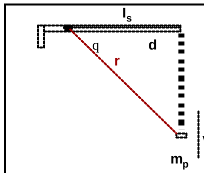


$I = m_p r^2$

$\omega = \frac{v_t}{r}$

v_t is the component of v perpendicular to r

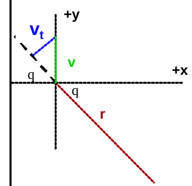
Does the angular momentum of the puck change as it moves toward the stick?



$I = m_p r^2$

$\omega = \frac{v_t}{r}$

$\cos\theta = \frac{d}{r}$



$\cos\theta = \frac{v_t}{v}$

$v_t = v\frac{d}{r}$

Angular momentum of puck

$$L = I\omega = \left(mr^2\right)\frac{v_t}{r} = mrv_t$$

$$L = mrv\frac{d}{r} = mvd$$

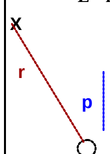
Does not change!!!

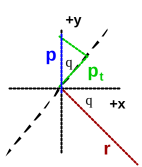
Angular Momentum as a Cross Product

$\vec{L} = I\vec{\omega}$ Angular momentum is in direction of angular velocity. Out in this case.

Another expression for angular momentum of a particle

$$\vec{L} = \vec{r} \times \vec{p}$$





Magnitude of angular momentum rp_t

$r(p\cos\theta)$

direction

right hand rule: from r to p

out

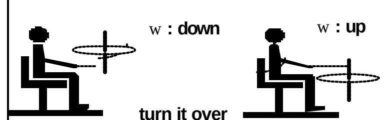
Conservation of angular momentum

$$\Delta\vec{L}_{system} = \Delta\vec{L}_{transfer}$$

Any change in the angular momentum of a system must come from

Interactions with objects outside the system

You choose the system



System bicycle wheel

Direction of Angular Momentum of system

Initial: down final: up

$$\int_0^{\text{interaction time}} \vec{\tau}\,dt = \Delta\vec{L}_{transfe}$$

Angular Momentum transfer by ?

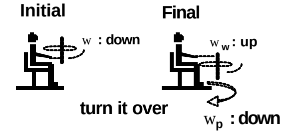
Another system

System bicycle wheel + person + stool

Chair is free to turn (no external torque)

No Angular Momentum transfer

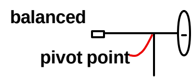
Initial Final



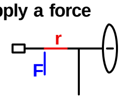
Direction of Angular Momentum of system

Initial: down Final: down

Angular Momentum Transfer



Now apply a force



The force causes a torque on the system

$$\vec{\tau} = \vec{r} \times \vec{F}$$

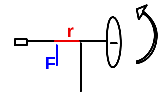
Torque direction: out

The torque causes angular momentum transfer to the system

$$\int_{\text{interaction time}} \vec{\tau}\,dt = \Delta\vec{L}_{transfer}$$

Angular momentum transfer direction: out

$\Delta L_{transfer}$ is out



$L_i = 0$ $L_f = ?$

Conservation of angular momentum

$$\vec{L}_f - \vec{L}_i = \Delta\vec{L}_{transfer}$$

$L_f = \Delta L_{transfer}$ L_f is out

$\vec{L}_f = I\vec{\omega}$ ω is out

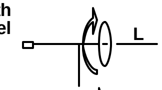
System turns up

What happens if wheel is spinning?

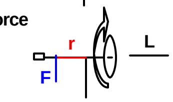
The rotating wheel has a large angular momentum

Angular momentum transfer

Balanced with rotating wheel



apply a force



The force causes a torque on the system

$$\vec{\tau} = \vec{r} \times \vec{F}$$

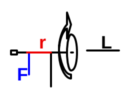
Torque direction: out

The torque causes angular momentum transfer to the system

$$\int_{\text{interaction time}} \vec{\tau}\,dt = \Delta\vec{L}_{transfer}$$

Angular momentum transfer direction: out

How does the system move?



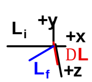
$\Delta L_{transfer}$ is out

Conservation of angular momentum

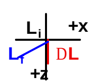
$$\vec{L}_f - \vec{L}_i = \Delta\vec{L}_{transfer}$$

$$\vec{L}_f = \Delta\vec{L}_{transfer} + \vec{L}_i$$

3-D view



top view



Angular momentum from the wheel is always along the bar

Bar turns so that wheel angular momentum in direction of final angular momentum

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When the bar turns

angular momentum transfer is still perpendicular to the bar

Bar turns again so that wheel angular momentum in direction of final angular momentum

The bar keeps turning

ω_{bar} is up

top view