CHAPTER 4: Results and Conclusions

This convergent study is the second part of a larger research program designed to understand physics instructors' conceptions about the teaching and learning of problem solving. Because the first part of the research program has set forth the foundation in this area as an exploratory study, this study was designed to be a more convergent study that would serve to critique and refine the initial explanatory model. The goal of this convergent study is to critique and refine the Problem-Solving Process part of the initial explanatory model. The refined explanatory model of the Problem-Solving Process is described by a concept map consisting of the type and range of conceptions held by 30 physics instructors that were interviewed. As discussed in Chapter 3, the main goal of this convergent study is to use a larger sample of higher education physics instructors to critique and refine the nature and range of physics instructors' conceptions about the problem-solving process that were generated during the previous, exploratory stage.

In this chapter I will use the three sub-questions as a way to guide the discussion. First I will discuss how the qualitatively different conceptions of the problem-solving process are refined in the Explanatory Model. These descriptions consist of the major components of the problem-solving process where a large percentage (> 30%) of the instructors view them in similar ways. Then I will discuss how the details of the qualitatively different conceptions of the problem-solving process are refined in the Explanatory Model. And finally, I will discuss whether the different conceptions of the problem-solving process are in reality qualitatively different.

Concept Map Symbols

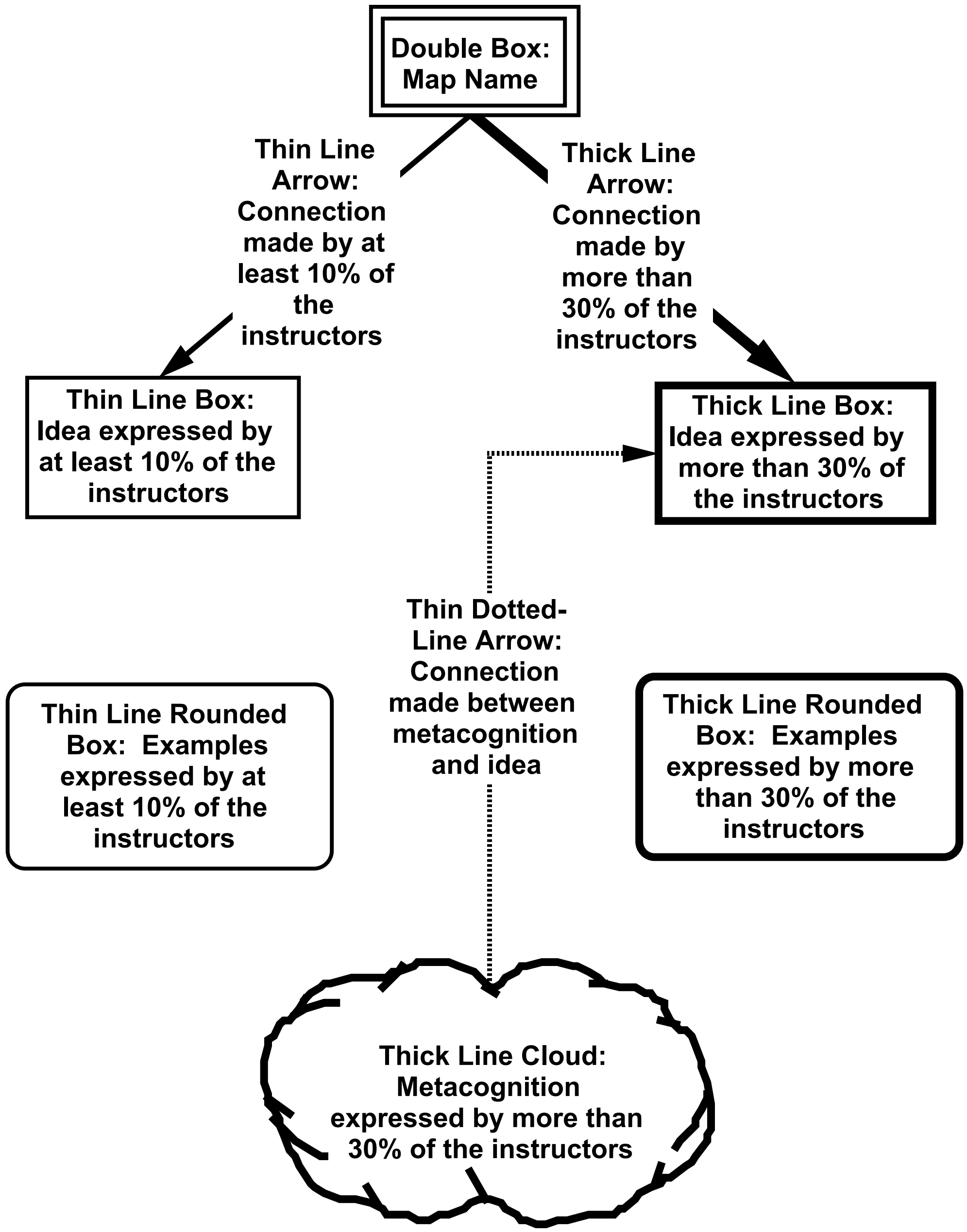
For this convergent study, only a few of the concept map symbols were necessary. The key for these symbols is presented in Figure 4-1, and the different symbols are briefly described below:

- **Double Box:** The double box contains the name of a feature of the explanatory model. In this convergent study, the feature is Solving Physics Problems.

- **Thin Line Box:** The thin line box represents an idea that was expressed by at least 10% of the number of instructors that expressed the views within a particular path.
- **Thick Line Box:** The thick line box represents an idea that was expressed by more than 30% of the number of instructors that expressed the views within a particular path.
- **Thin Line Rounded Box:** The thin line rounded box represents examples of an idea that was expressed by at least 10% of the number of instructors that expressed the views within a particular path.
- **Thick Line Rounded Box:** The thick line rounded box represents examples of an idea that was expressed by more than 30% of the number of instructors that expressed the views within a particular path.
- **Thin Line Arrow:** The thin line arrow connecting two boxes represents a relationship that was explicitly expressed by at least 10% of the number of instructors that expressed the views within a particular path.
- **Thick Line Arrow:** The thick line arrow connecting two boxes represents a relationship that was explicitly expressed by more than 30% of the number of instructors that expressed the views within a particular path.
- **Thick Line Cloud:** The thick line cloud represents examples of metacognition that was expressed by more than 30% of the number of instructors that expressed the views within a particular path.
- **Thin Dotted-Line Arrow:** The thin dotted-line arrow connects metacognition to the ideas in the path.

In order to allow the readers to make their own judgments of the level of empirical support for each part of the problem-solving process, each box contains information about the percentage of instructors within each conception of the problem-solving process that expressed that particular idea during the interview.

Figure 4-1: Concept Map Symbols



Refining the Explanatory Model of the Problem-Solving Process

The Initial Explanatory Model indicated that there are probably three qualitatively different conceptions of the problem-solving process: (1) A linear decision-making process; (2) A process of exploration and trial and error; and (3) An art form that is different for each problem. The research question for this convergent study is:

To what extend does the Initial Explanatory Model of instructors' conceptions about the problem solving process need refinement and expansion?

To answer the research question, there are consequently, and logically, three sub-questions to be answered. The following sections will address each of these three sub-questions in sequence.

Sub-Question 1: Qualitatively Different Conceptions of the Problem-Solving Process

This section will discuss the results pertaining to the first sub-question for this convergent study. The first sub-question for this convergent study is: When the sample of instructors is increased from 6 to 30,

Do the three qualitatively different conceptions of the problem-solving process in the Initial Explanatory Model remain the same?

To answer this sub-question, 24 additional interviews with physics instructors from other types of higher education institutions were analyzed. The resulting 24 individual concept maps, along with the 6 from the initial model, were combined to form a new composite map that serves as the Refined Explanatory Model of instructors' conceptions about the problem-solving process. The model is shown in Figure 4-2. The major components of the qualitatively different conceptions are described below, and also summarized in Table 4-1. All 30 instructors described the conception that the process of solving physics problems can be characterized as set of decisions that needs to be made.

<u>Overview of the Qualitatively Different Conceptions in the Initial Explanatory Model</u>

The initial explanatory model of instructors' conceptions about the problem-solving process was developed from analyzing the interviews with six research university physics instructors. All six instructors expressed the similar conception that the process of solving physics problems requires using an understanding of PHYSICS CONCEPTS and SPECIFIC TECHNIQUES. The three qualitatively different ways that these six instructors characterized the problem-solving process are a linear decision-making process, a process of exploration and trial and error, and an art form that is different for each problem. Each instructor described only one conception of the problem-solving process.

1. ***A linear decision-making process.*** Problem solving is a linear decision-making process where PHYSICS CONCEPTS and SPECIFIC TECHNIQUES are used in a complicated way to determine what to do next. From this point of view, problem solving involves making decisions, and the correct decision is always made. There is no need to backtrack. The three instructors with this conception of problem solving expressed varying degrees of detail about the problem-solving process. All of these conceptions, however, are vague. For example, even though these instructors all said that an important step in the problem-solving process was deciding on the physics principles, none clearly explained how this was done.

2. ***A process of exploration and trial and error.*** Problem solving is a process where an understanding of PHYSICS CONCEPTS is used to explore and come up with possible choices that are then tested. The conception recognizes that making mistakes and having to backtrack is a natural part of problem solving. Although these instructors were able to describe the problem-solving process in more detail than those in the previous group, there were still aspects that were not fully explained. For example, the instructors seemed unclear about how a student should come up with possible choices to try. The instructors seemed to think that it involved more than random guessing from all of the concepts that had been learned in the class, but did not articulated how an understanding of PHYSICS CONCEPTS was used to come up with possible choices.

3. ***An art form that is different for each problem.*** Problem solving is artfully crafting a unique solution for each problem. This one instructor did not provide any details about how one should go about doing this.

Qualitatively Different Conceptions in the Refined Explanatory Model

There are again three qualitatively different ways that the physics instructors in this convergent study characterized the process of solving physics problems: a decision-making process that is linear, a decision-making process that is cyclical, and a decision-making process that is artistic. Similar to the initial explanatory model, each instructor described only one of these three qualitatively different conceptions of the problem-solving process.

1. ***A decision-making process that is Linear.*** 22 of the 30 physics instructors described problem solving as a decision-making process that is "Linear". On a global scale, descriptions here are similar to those from the initial explanatory model, and nothing is unexpected. The process involves the problem solver to first understand the problem. And with visualization, extraction, and categorization information from the problem situation (such as listing, labeling, and defining variables, and drawing pictures and diagrams), the problem solver can then make decisions on where to start the solution from having an understanding of general physics principles and concepts. Once having recognized and decided on the principles and concepts that are needed to solve the problem, the problem solver can then simply apply them to get the answer. And finally, the problem-solving process is completed when the problem solver checks the unit and evaluates the reasonableness of the answer to see that it is correct. From this point of view, problem solving involves making decisions, and the correct decision is always made. There is no need to backtrack. The 22 instructors with this view of problem solving expressed varying degrees of detail about different parts of the process. These details will be discussed in a later section.

2. ***A decision-making process that is Cyclical.*** 7 of the 30 physics instructors described problem solving as a decision-making process that is "Cyclical". The descriptions here are an expansion of the "Exploration and Trial and Error" view of the problem-solving process in the initial explanatory model. The descriptions of this view explicitly reflect these instructors' recognition that problem solving naturally requires progress checking. It is also natural, and often necessary, to go back and redo a previous step after having made a mistake while solving a problem. The process first involves understanding, focusing, visualizing, and analyzing of the problem (such as by drawing pictures and diagrams). Then the problem solver needs to brainstorm and explore to come up with possible approaches to solve the problem, and that requires having an understanding of general physics principles and concepts. The next step in the process is to experiment on an approach by figuring out what information is needed and solve for what is being asked in the problem. This is the step during which the problem solver would apply the principles and concepts. At this point if the problem solver realizes that the solution does not work, the problem solver would have to go back to brainstorm and explore to come up with other possible approaches. Having gone through the mathematics to get an answer, the potential final step in the solution process is to evaluate the answer (such as by checking the units and the reasonableness of the answer). It is the potential final step because these instructors also described the possibility that if the evaluation resulted in the realization that the answer is not correct, the problem solver would then need to go back again to brainstorm and explore. From this point of view, problem solving also involves making decisions, but the correct decision is not always made. There is an explicit recognition of the need to "go back" to a previous step when a mistake is spotted through checking the solution, both during the process and at the end of the solution. The 7 instructors with this view of problem solving all expressed varying degrees of detail about different parts of the process. These details will be discussed in a later section.

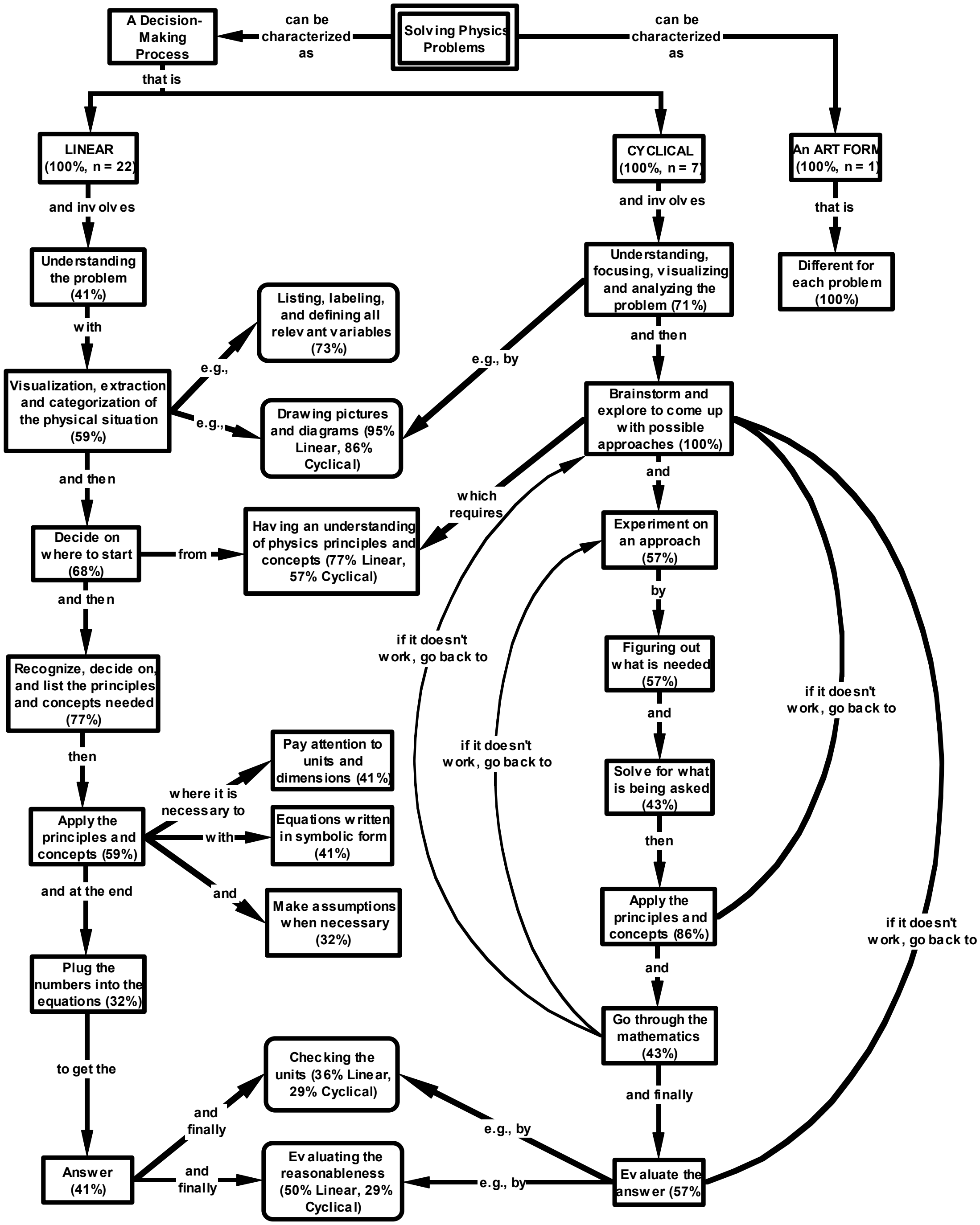
3. ***An Art Form that is different for each problem.*** One instructor in the initial explanatory model described problem solving as artfully crafting a unique solution for each problem. This instructor did not provide any details about how a problem solver would go about doing this. No instructor in the expanded sample described the problem-solving process in this fashion.

These 30 physics instructors characterized the problem-solving process in three qualitatively different ways. Since the third way lacked any description of a process, it consequently cannot be compared and contrasted with the other two in more detail. Although the linear and cyclical characterizations of the problem-solving process, heretofore denoted as *Linear* and *Cyclical*, shared some similarities in their major components, they differed in their descriptions of how these components are pertinent to a successful problem solution.

Table 4-1: Summary of the qualitatively different conceptions of the problem-solving process

<table>
<tr><th rowspan="2">Conceptions of the Problem-Solving Process</th><th colspan="4">Polya's Problem-Solving Steps</th></tr>
<tr><th>Understanding the Problem</th><th>Making a Plan</th><th>Carrying Out the Plan</th><th>Looking Back</th></tr>
<tr><td rowspan="2">Decision-Making Process that is LINEAR</td><td>• Visualize, Extract, and Categorize information from the problem statement
• List, Label, and Define variables
• Draw pictures and diagrams</td><td>• Make decision on where to start based on having an understanding of the general physics principles and concepts
• Recognize and decide on the principles and concepts that are needed to solve the problem</td><td>• Apply the principles and concepts to get the answer</td><td>• Check the units of the answer
• Evaluate the reasonableness of the answer</td></tr>
<tr><td colspan="4">Correct decision is always made based on having the understanding of the general physics principles and concepts; therefore, no backtracking is necessary when solving a problem.</td></tr>
<tr><td rowspan="2">Decision-Making Process that is CYCLICAL</td><td>• Understand, Focus, Visualize, and Analyze the problem
• Draw pictures and diagrams</td><td>• Brainstorm and Explore to come up with possible approaches to solve the problem based on having an understanding of the general physics principles and concepts
• Decide on what approach to experiment</td><td>• Experiment on an approach by figuring out what information is needed and solve for what is being asked in the problem
• Apply the principles and concepts
• If the solution does not progress, go back to the previous step and come up with other possible approaches
• Go through the mathematics to get an answer</td><td>• Evaluate the answer
• Check the units and the reasonableness of the answer
• If the evaluation resulted in the realization that the answer is not correct, go back to brainstorm and explore other possible approaches</td></tr>
<tr><td colspan="4">Problem solving naturally requires progress checking, because the correct decision is not always made; therefore, it is often necessary to go back and redo a previous step after having made a mistake.</td></tr>
<tr><td>An ART FORM that is different for each problem</td><td colspan="4">No descriptions given of a process.</td></tr>
</table>

Figure 4-2: Refined Explanatory Model – Problem-Solving Process (30 Instructors)



Refined Explanatory Model: Answers to Sub-Question 1

The first sub-question for this convergent study is: When the sample of instructors is increased from 6 to 30,

Do the three qualitatively different conceptions of the problem-solving process in the Initial Explanatory Model remain the same?

The three qualitatively different conceptions of the problem-solving process identified in the Initial Explanatory Model underwent some changes when the sample of instructors was expanded from 6 to 30. Table 4-2 provides a summary of the findings.

Summary of Conception 1

The "Linear Decision-Making Process" conception identified in the Initial Explanatory Model remained as the "Decision-Making Process that is Linear" conception in the Refined Explanatory Model. In the initial model, idiosyncrasies in the order of some of the components within the problem-solving process existed. This put the sequencing of the decisions that needed to be made in problem solving into question. The sample was not large enough to determine whether a particular sequence is more representative of the instructors than the other. With the expansion of the sample, the sequencing issue was able to be addressed. The results show that there is clearly a sequence of the linear conception in the refined model that is more representative of the sample. This resulted in the first qualitatively different conception of problem-solving: ***A decision-making process that is Linear***.

Summary of Conception 2

The "Exploration and Trial and Error" conception identified in the Initial Explanatory Model became the "Decision-Making Process that is Cyclical" conception in the Refined Explanatory Model. In the initial model, there was only one instance of the necessary backtracking from one step to a previous step during problem solving. With the expanded sample, the necessity of backtracking became more concrete, and involved going from multiple steps to multiple previous steps. In the refined model, the ideas of exploration and experimentation of approaches continued to be well supported. In addition, with the expanded sample, the idea that problem solving is naturally iterative also became apparent. This resulted in the second qualitatively different conception of problem-solving: ***A decision-making process that is Cyclical***.

Summary of Conception 3

The idiosyncratic conception that problem solving is "An Art Form that is different for each problem" identified in the Initial Explanatory Model remained as an idiosyncratic conception in the Refined Explanatory Model. No other instructor in the expanded sample conceived of the problem-solving process in the same fashion.

To sum up, this convergent study found that the Explanatory Model of the Problem-Solving Process consists of two qualitatively different conceptions; a decision-making process that is Linear, and a decision-making process that is Cyclical. The third conception remained idiosyncratic and with no descriptions of a process, and will no be included in the model. The rest of this chapter will consequently only discuss the Linear and Cyclical characterizations of the problem-solving process.

Table 4-2: Comparisons of the qualitatively different conceptions of the problem-solving process

Qualitatively Different Conceptions of the Problem-Solving Process	**Initial Explanatory Model (Exploratory Study)**	**Refined Explanatory Model (Convergent Study)**
1	*Linear Decision-Making Process*	*Decision-Making Process that is Linear*
2	*Process of Exploration and Trial and Error*	*Decision-Making Process that is Cyclical*
3	*An Art Form that is different for each problem*	*An Art Form that is different for each problem (Dropped)*

Sub-Question 2: Details in the Refined Explanatory Model

This section will discuss the results pertaining to the second sub-question for this convergent study. The second sub-question for this convergent study is: When the sample of instructors is increased from 6 to 30,

Where appropriate, can the lack of detail in the problem-solving process be filled?

The Refined Explanatory Model (Figure 4-2) illustrates the similar ideas of the Problem-Solving Process that at least 30% of the instructors within each qualitatively different conception had about the problem-solving process. Different instructors, however, sometimes expressed some of the components in different ways and in differing amounts of details. The ideas expressed by less than 30% of the instructors were not illustrated on this map.

Another detail in the Refined Explanatory Model of the Problem-Solving Process is the descriptions of the role of metacognition in the problem-solving process. As discussed in Chapter 2, metacognition was defined as “knowledge and cognition about cognitive phenomena” (Flavell, 1979, p. 906). In other words, it is simply the thinking about ones own thinking. In relation to problem solving, research has shown that successful problem solvers not only spend more time analyzing a problem and the directions that may be taken than less successful students, but also monitor and assess their actions and cognitive processes throughout the problem-solving process (Lester et. al., 1989; Schoenfeld, 1983, 1985a, 1985b, 1987). Other research (see for example Paris & Winograd, 1990) has also provided evidence that metacognition helps to orchestrate aspects of problem solving, including the processes of making plans before tackling a task (*Planning*), making adjustments while working on a task (*Monitoring*), and making revisions after having worked on a task (*Evaluation*).

The following sections will describe each of the two qualitatively different conceptions of the refined explanatory model in more detail one by one. The sections for each qualitatively different conception will include first a discussion about the details of the major components, and then a discussion of the role of metacognition (Linear Conception, Figure 4-3; Linear Conception with Metacognition, Figure 4-4; Cyclical Conception, Figure 4-5, Cyclical Conception with Metacognition, Figure 4-6). For ease of reference, the details will be *italicized* within quotation marks, the major components will be **bolded**, and the metacognitions will be <u>*italicized and underlined*</u> in the following discussions.

<u>Details of the Major Components in the Linear Conception</u>

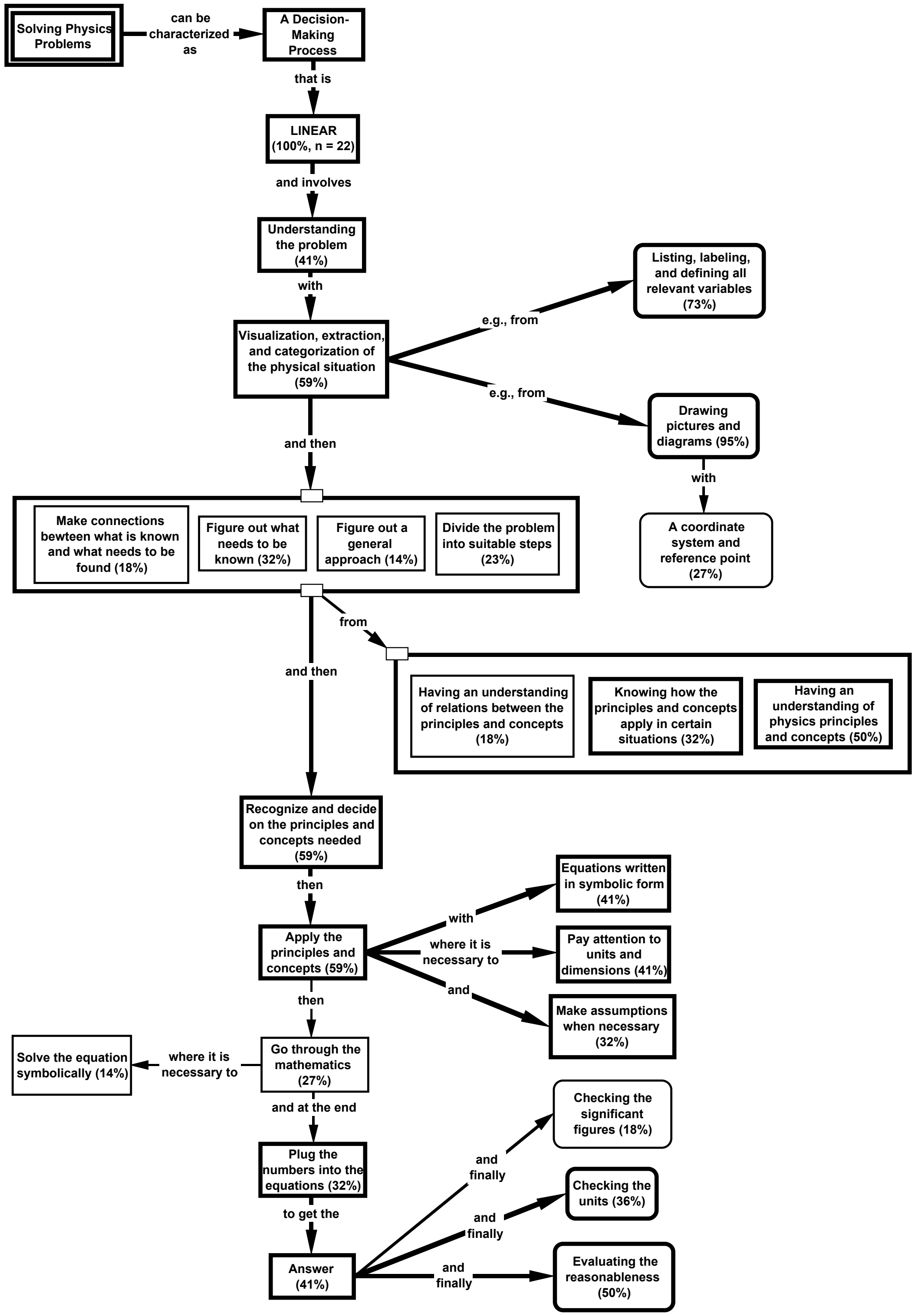
The 22 physics instructors that expressed this Linear conception of the problem-solving process mostly had similarly vague descriptions of the major components of the process. There were two components that were described in different ways and in slightly more detail. The first of which was the component of **decide on where to start**; 68% of the 22 instructors that expressed the Linear conception described this component as the step immediately after **visualization, extraction, and categorization of the physical situation**. Out of these instructors, 20% expressed this component as a general description of "*figure out a general approach*". Other instructors expressed this component in terms of more specific actions; 27% of them stated the need to "*make connections between what is known and what needs to be found*", 33% of them stated the need to "*divide the problem into suitable steps*", and 47% of them stated the need to "*figure out what needs to be known*". Three of these instructors expressed multiple descriptions, and that resulted in the sum of the percentages to be over one hundred percent.

The second component that was described in different ways was **having an understanding of physics principles and concepts**; 77% of the 22 instructors that expressed the Linear conception explicitly described this component as a necessary element of the problem-solving process. Out of these instructors, 65% expressed this component in the same holistic wording as the component. Other instructors expressed this component in slightly different ways; 24% of them stated the necessity of "*having an understanding of relations between the principles and concepts*", and 32% of them expressed the necessity of "*knowing how the principles and concepts apply in certain* *situations*". Four of these instructors expressed multiple descriptions, and that again resulted in the sum of the percentages to be over one hundred percent.

Of the instructors that expressed the previous component of **decide on where to start**, 80% of them also expressed the necessity of "*having an understanding of physics principles and concepts*" to facilitate that decision. Out of these instructors, 58% expressed this component in the same holistic wording as the component, 33% of them stated the necessity of "*having an understanding of relations between the principles and concepts*", and 42% of them expressed the necessity of "*knowing how the principles and concepts apply in certain situations*". Again, four of these instructors expressed multiple descriptions, and that resulted in the sum of the percentages to be over one hundred percent.

There were some less common ideas that did not make the 30% cutoff. Of the 95% of the instructors that described **drawing pictures and diagrams**, about one out of four, which constitutes 27% of the instructors that expressed the Linear conception, also included "*a coordinate system and referent point*". Some instructors described the step of "*go through the mathematics*" in between **apply the principles and concepts** and **plug the numbers into the equations**. Although this seems to be an obvious step, only about one out of four of the instructors that expressed the Linear conception explicitly mentioned it. Another 14% of the instructors expressed the necessity to "*solve the equation symbolically*" before one could **plug the numbers into the equations**. Still another 18% of the instructors described the step of "*checking the significant figures*" of the **answer**. Although these ideas were less common than those included in the major components map (Figure 4-2), they nonetheless represent information relevant to the problem-solving process. And due to the small numbers in the sample, it is difficult to determine whether these less common ideas are indeed idiosyncratic or not.

Figure 4-3: More detailed concept map for the Linear Decision-Making Process conception



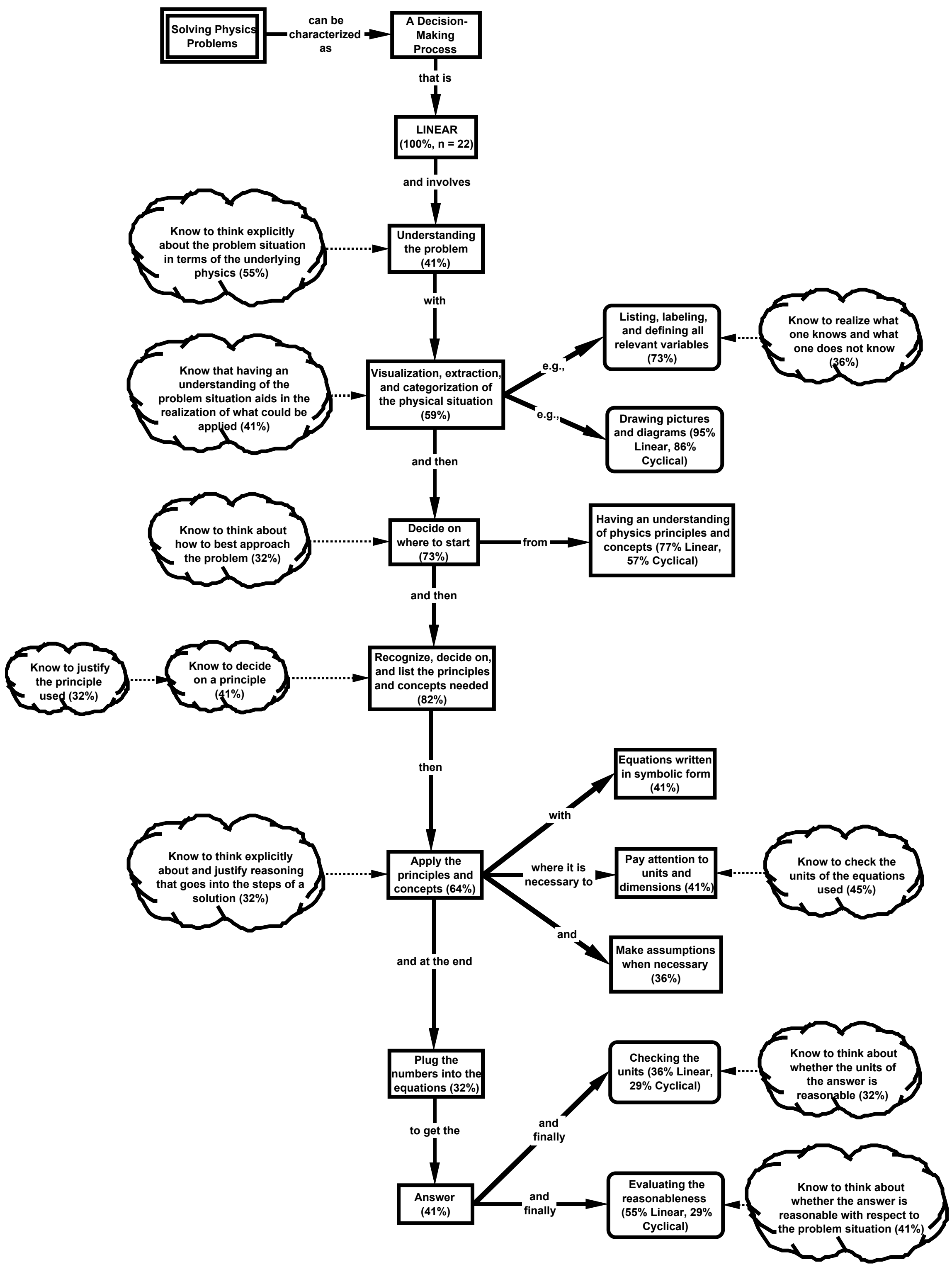
Metacognition in the Linear Conception

The 22 instructors that expressed the Linear conception of the problem-solving process also expressed 10 different metacognitions that underlie the process. Figure 4-4 is a reproduction of the Linear conception from Figure 4-3, with the addition of the metacognitions connected to the relevant components. It is interesting to note here that the majority of the components only had one metacognition linked to them. The metacognitions will be described in the sequence of the Linear conception of the problem-solving process.

These instructors expressed the necessity of the problem solver to *know to think explicitly about the problem situation in terms of the underlying physics* in order to **understand the problem**, because the problem solver also need to *know that having an understanding of the problem situation aids in the realization of what could be applied*. In addition, the problem solver also needs to *know to realize what one knows and what one does not know* when **listing, labeling, and defining all relevant variables**. When talking about the problem-solving component of **deciding on where to start**, the instructors expressed the need to *know to think about how to best approach the problem*, and then *know to decide on a principle* to be used, and also *know to justify the principle*. The instructors went on to described the need to *know to think explicitly about and justify reasoning that goes into the steps of a solution* when the problem solver is **applying the principles and concepts** that has been decided upon. It is also necessary, at this stage, to *know to check the units of the equations used*. Finally, after having reached the **answer**, the problem solver needs to not only *know to think about whether the units of the answer is reasonable*, but also *know to think about whether the answer is reasonable with respect to the problem situation*.

The above description should not come as a surprise to anyone, and the metacognitions were all reasonably connected to the relevant major components of the problem-solving process. There were, however, a few noticeable omissions. First, the instructors did not express any metacognition in relation to the major component of **drawing pictures and diagrams**. Second, the instructors did not express any metacognition in relation to the major component of **having an understanding of physics principles and concepts**, which allows the problem solver to **decide on where to start**. Third, although a large percentage of the instructors expressed the need to **make assumptions when necessary**, no one described any metacognition that underlie the process.

Figure 4-4: Linear Decision-Making Process concept map with Metacognition



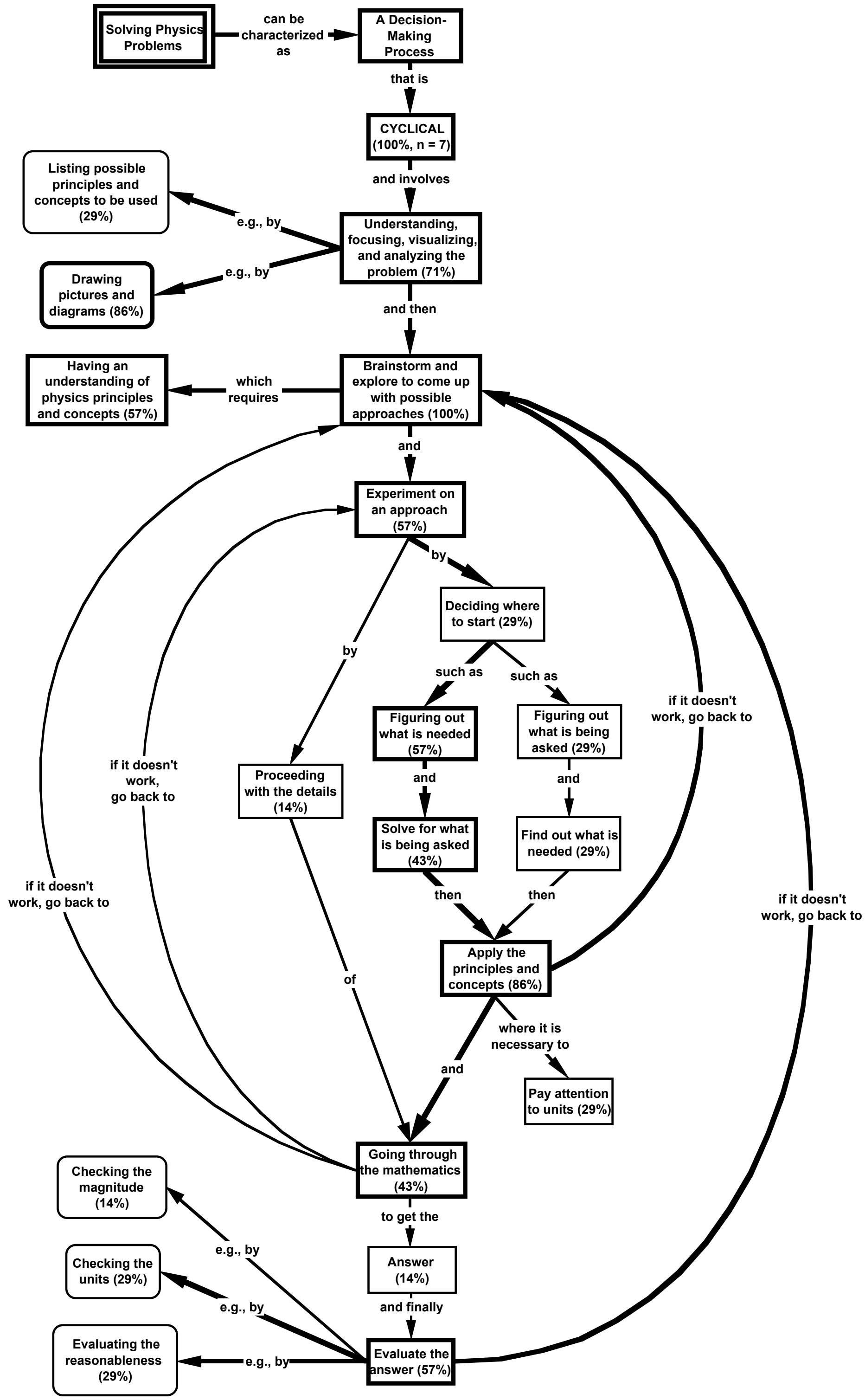
Details of the Major Components in the Cyclical Conception

The 7 physics instructors that expressed this Cyclical conception of the problem-solving process had similarly vague descriptions of the major components of the process. Unlike in the Linear conception, there were no major components in the Cyclical conception that had different descriptions by different instructors.

Some of the less common ideas in this Cyclical conception were almost at the 30% cutoff. First of all, 29% of the instructors that expressed this conception of problem solving talked about the **understanding, focusing, visualizing, and analyzing the problem** component in terms of "*listing possible principles and conceptions to be used*". This is in addition to **drawing pictures and diagrams**. In terms of the steps of **experiment on an approach**, 29% of the instructors described the need to "*decide where to start*" by "*figuring out what is being asked*" and then "*find out what is needed*". This is in the opposite order of the sequence in Figure 4-2, where a larger percentage of the instructors described the steps as first **figuring out what is needed** and then **solve for what is being asked**. In describing the problem-solving component of **apply the principles and concepts**, 29% of the instructors also expressed the necessity to "*pay attention to units*".

There were also some ideas that were more idiosyncratic. In describing the intermediate link between the problem-solving components of **experiment on an approach** and **going through the mathematics**, 14% of the instructors vaguely expressed it as "*proceeding with the details*". It was unclear if "*proceeding with the details*" was another way of describing the mathematics, or other steps prior to **going through the mathematics**. Although 57% of the instructors that expressed this Cyclical conception of problem solving mentioned the necessary component of **evaluate the answer**, only 14% explicitly mentioned anything about getting the "*answer*" as part of the process. Another 14% of the instructors described "*checking the magnitude*" as a way to **evaluate the answer**.

Figure 4-5: More detailed concept map for the Cyclical Decision-Making Process conception



Metacognition in the Cyclical Conception

The 7 instructors that expressed the Cyclical conception of the problem-solving process also expressed 18 different metacognitions that underlie the process. Figure 4-6 is a reproduction of the Cyclical conception from Figure 4-5, with the addition of the metacognitions connected to the relevant components. It is interesting to note here, in contrast to the Linear conception, that some of the major components had multiple metacognitions linked to them. The metacognitions will again be described in the sequence of the Cyclical conception of the problem-solving process.

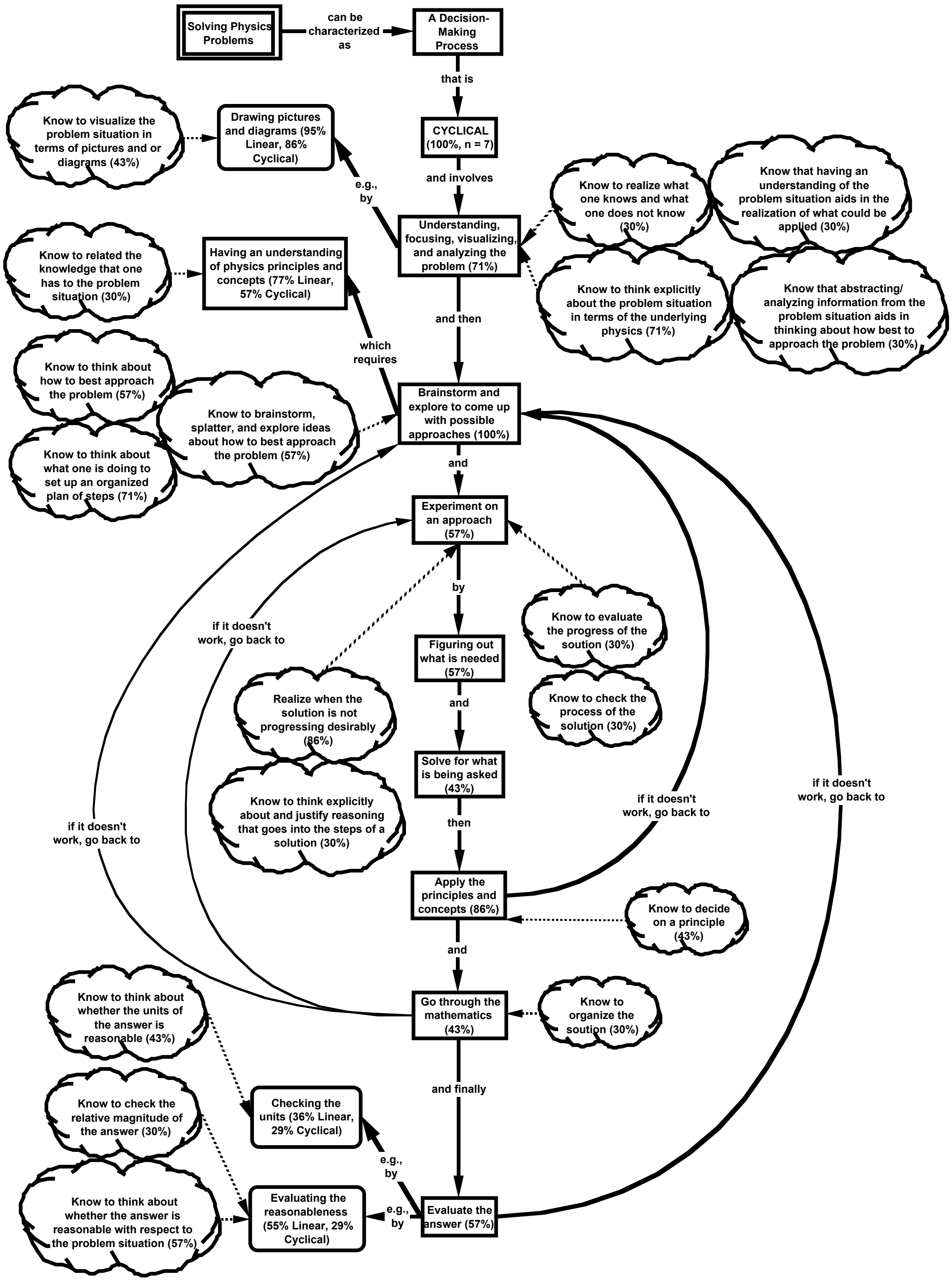
These instructors expressed the first step of problem solving as **understanding, focusing, visualizing, and analyzing the problem**. In order to do this, the problem solver need to *know to think explicitly about the problem situation in terms of the underlying physics*, *know that having an understanding of the problem situation aids in the realization of what could be applied*, and *know to realize what one knows and what one does not know*. In addition, instructors in the Cyclical conception expressed the necessity for the problem solver to *know that abstracting/analyzing information from the problem situation aids in thinking about how best to approach the problem*, and that *knowing to visualize the problem situation in terms of pictures and/or diagrams* helps one **draw pictures and diagrams**, which in turn helps the problem solver with **understanding, focusing, visualizing, and analyzing the problem**.

When describing the need for problem solvers to **brainstorm and explore to come up with possible approaches**, these instructors expressed the metacognitions of *know to think about how to best approach the problem*, *know to think about what one is doing to set up an organized plan of steps*, and *know to brainstorm, splatter, and explore ideas about how to best approach the problem*. These metacognitions, along with *knowing to relate the knowledge that one has to the problem situation* that is linked to **having an understanding of physics principles and concepts**, lead the problem solver to **experiment on an approach**. In the overall scope of the experimentation process, these instructors expressed several metacognitions that facilitate the “go back” paths that make the Cyclical conception cyclical. These metacognitions are *know to think explicitly* *about and justify reasoning that goes into the steps of a solution*, *know to evaluate the progress of the solution*, *know to check the process of the solution*, and *realize when the solution is not progressing desirably*. Within the steps of the experimentation process, the problem solver also needs to *know to decide on a principle* in order to **apply the principles and concepts**, and *know to organize the solution* when **going through the mathematics**.

Finally, in **evaluate the answer**, these instructors expressed the necessary metacognitions of *know to think about whether the units of the answer are reasonable*, *know to check the relative magnitude of the answer*, and *know to think about whether the answer is reasonable with respect to the problem situation*.

Again, the above description should not come as a surprise to anyone, and the metacognitions were all reasonably connected to the relevant components of the problem-solving process. One noticeable detail of this conception is that these instructors expressed some sort of metacognition for every major component of the problem-solving process that was described.

Figure 4-6: Cyclical Decision-Making Process concept map with Metacognition



Metacognition in the Problem-Solving Process

This section will first discuss how the physics instructors in this convergent study, overall, expressed the three different types of metacognition in relation to problem solving. Then comparisons of the role of metacognition will be made between the two qualitatively different conceptions: Linear and Cyclical. The comparisons will be made first with respect to the percentage of statements within each qualitatively different conception that describes the different types of metacognition, then with respect to the ways that the different types of metacognition were phrased within each qualitatively different conception.

Different Types of Metacognition

As mentioned earlier, there are three types of metacognition (*planning, monitoring, evaluation*) that help to orchestrate different aspects of problem solving, and the relevant instructor statements were categorized as such. Metacognitive statements related to starting a solution to a problem were coded as *planning* statements. Metacognitive statements related to checking the progress of a solution to a problem were coded as *monitoring* statements. Metacognitive statements related to checking the reasonableness of a solution to a problem were coded as *evaluation* statements. Table 4-3 provides the summary of results for all 30 instructors in the sample. The table provides, for each instructor, the count for total number of problem solving statements, total number of metacognitive statements, and the number for each of the three types of metacognition.

A naïve assumption could be that these instructors, experts in their ability to solve problems in physics, would consider *planning*, *monitoring*, and *evaluation* equally in problem solving. Whether they explicitly recognize these as metacognitions, the instructors should more or less express these notions equally when describing the problem-solving process during the interview. Therefore, the number of statements made about the three types of metacognition would consequently be equal. A χ^2-test was performed to determine the significance of this null hypothesis ($\mu_p = \mu_m = \mu_e$, k = 3, df = 2). Using the functions in Excel®, the statistical analysis yielded a value of $\chi^2 = 209.15$

($p < 0.000$). Therefore it is apparent that these 30 physics instructors, as a whole, did not talk about the three types of metacognition equally when describing the problem-solving process during the interviews.

As a matter of fact, a quick view at the numbers in Table 4-3 would lead one to make certain alternative claims about these 30 physics instructors: 1) these instructors made significantly more statements about the metacognition of *planning* than *monitoring* and *evaluation*; 2) these instructors made more than twice as many statements about the metacognition of *planning* than *monitoring*; and 3) these instructors made almost 5 times as many statements about the metacognition of *planning* than *evaluation*. These findings seem to indicate that, for these instructors, once the *planning* is complete, the problem is more or less solved.

Table 4-3: Summary of number and type of statements made by each of the 30 physics instructors

Instructor #	Number of Statements				
	Problem Solving	**Metacognition**	**Planning**	**Monitoring**	**Evaluation**
1	61	20	10	10	0
2	47	21	17	4	0
3	115	36	18	10	8
4	111	31	17	14	0
5	87	27	17	6	4
6	96	32	20	11	1
7	95	35	17	10	8
8	65	15	12	3	0
9	49	7	5	2	0
10	88	30	17	9	4
11	64	22	12	5	5
12	41	8	6	2	0
13	43	12	6	3	3
14	40	12	7	1	4
15	69	18	7	5	6
16	66	22	15	7	0
17	72	17	11	3	3
18	64	16	7	5	4
19	77	23	10	7	6
20	50	9	4	3	2
21	66	24	15	6	3
22	116	53	31	21	1
23	82	22	16	5	1
24	90	26	15	6	5
25	66	19	12	6	1
26	29	10	7	0	3
27	26	8	6	2	0
28	37	14	10	4	0
29	22	10	10	0	0
30	14	7	3	2	2
Min	***14***	***7***	***3***	***0***	***0***
Max	***116***	***53***	***31***	***21***	***8***
Average	***65***	***20***	***12***	***6***	***2***
Total	***1948***	***606***	***360***	***172***	***74***

Comparison of Metacognition between the Linear and Cyclical Conceptions

This section will discuss the similarities and differences in the role of metacognition in the problem-solving process as described by the different instructors who expressed the two qualitatively different conceptions. The comparison will be two fold; the percentage of statements about the three types of metacognition (*planning*, *monitoring*, *evaluation*) made during the interview, and the percentage of instructors who made the various different phrasings of metacognition.

Percentage of statements

Instructors made metacognitive statements when describing the problem-solving process during the interviews. As mentioned earlier, these statements can be further divided into three different types of metacognition. The percentage of metacognitive statements with respect to the total problem solving statements, and the percentage of each of the three types of metacognition with respect to the total problem solving statements are shown in Table 4-4 for the Linear conception and Table 4-5 for the Cyclical conception.

The first comparison is in the overall percentage of metacognitive versus problem solving statements. In the Linear conception, the distribution of the percentages is primarily in the 20% and 30% range, with an average of 29%. In the Cyclical conception, the distribution of the percentages is primarily in the 30% and 40% range, with an average of 39%. The next comparison is in the average percentages of each of the three different types of metacognition. In the Linear conception, the 29% was distributed across *planning*, *monitoring*, *evaluation* at 18%, 7%, and 4%, respectively. In the Cyclical conception, the 39% was distributed across *planning*, *monitoring*, *evaluation* at 24%, 11%, and 4%, respectively. Chart 4-1 illustrates these results graphically. Looking strictly at the numbers, both groups of instructors reflected a similar trend in the way they expressed these three different types of metacognition. The metacognitive statements about *planning* were expressed a larger percentage of time, on average, than statements about *monitoring* and *evaluation*. This is consistent with the result of the χ^2-test for the whole sample reported earlier.

Chart 4-1: Comparison of the percentages for the three different types of metacognition between the Linear and Cyclical conceptions

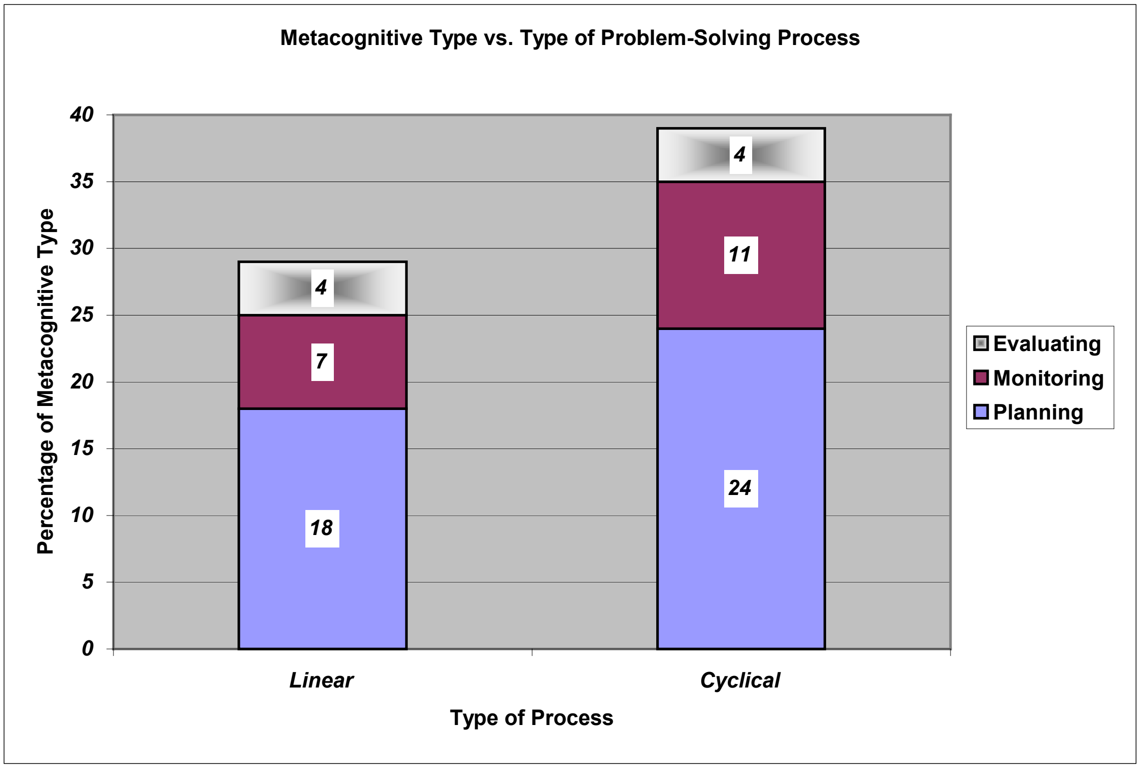


Table 4-4: Percentage of each type of statement with respect to the total number of problem solving statements for each instructor that expressed the Linear conception (N = 22)

Instructor #	Percentage (%) of Statements with respect to Problem Solving				
	Conception	Metacognition	Planning	Monitoring	Evaluation
1	***Linear***	33	16	16	0
3		31	16	9	7
4		28	15	13	0
7		37	18	11	8
8		23	18	5	0
9		14	10	4	0
10		34	19	10	5
11		34	19	8	8
12		20	15	5	0
13		28	14	7	7
14		30	18	3	10
15		26	10	7	9
16		33	23	11	0
17		24	15	4	4
18		25	11	8	6
19		30	13	9	8
20		18	8	6	4
23		27	20	6	1
24		29	17	7	6
26		34	24	0	10
27		31	23	8	0
29		45	45	0	0
Average		***29***	***18***	***7***	***4***

Table 4-5: Percentage of each type of statement with respect to the total number of problem solving statements for each instructor that expressed the Cyclical conception (N = 7)

Instructor #	Percentage (%) of Statements with respect to Problem Solving				
	Conception	Metacognition	Planning	Monitoring	Evaluation
2	***Cyclical***	45	36	9	0
5		31	20	7	5
6		33	21	11	1
21		36	23	9	5
22		46	27	18	1
25		29	18	9	2
30		50	21	14	14
Average		***39***	***24***	***11***	***4***

Different Phrasings of Metacognition

This comparison is different from the previous sections. The previous discussions about metacognition only involved those that were expressed by at least 30% of the instructors in each of the qualitatively different conceptions. This comparison of the descriptions will include all of the different metacognitions that were expressed by the instructors. In this comparison, it is useful to discuss the three different types of metacognition separately. As a reminder to the reader, every metacognitive statement made by each individual instructor during the interview was sorted into one of the three types of metacognition: *planning*, *monitoring*, and *evaluation*. Within each type of metacognition, instructors' statements were then categorized into groups based on idea similarities in the statements. A new phrasing that reflected the idea for that particular metacognition was then developed. These new metacognitive phrasings were then compared with the original instructor statements within each category to check for reasonableness of the rephrasing. The original wording from the instructor statements was kept as much as possible to limit over-extending the interpretations.

Planning involves metacognitions that are related to starting a solution to a problem. Table 4-6 shows the list of all of the *planning* phrases that at least 10% of the 29 instructors expressed (as mentioned earlier in the chapter, 1 of the 30 instructors in the sample expressed a Artistic conception of the problem-solving process without describing a process, therefore has necessarily been left out of any of the comparisons). The metacognitive phrases are grouped based on the similarities in their relations to particular components of the problem-solving process. The metacognitive phrases are also sequenced in the way that the instructors in the problem-solving process sequence the major components.

There are similarities in the way the two groups of instructors described some of the metacognitions. Large percentages of both the Linear and Cyclical instructors expressed the metacognition of: 1) *know to think explicitly about the problem situation in terms of the underlying physics*; 2) *know that having an understanding of the problem situation aids in the realization of what could be applied*; 3) *know to think about how to* *best approach the problem*; 4) *know to realize what one knows and what one does not know*; and 5) *know to decide on a principle*. (It is necessary here to indicate that the researcher deemed it appropriate to include a metacognition as characteristic of the Cyclical conception when at least 29% of the instructors expressed it.) It is not surprising that physics instructors agree that these five metacognitions are necessarily the underlying thought processes at various key steps within the planning stage of the problem-solving process. Some instructors expressed other necessary metacognitions.

Some metacognitions that were characteristic for one conception were not expressed by a large enough percentage of the instructors in the other conception to also be considered as characteristic. The most apparent of which are the need to: 1) *know to visualize the problem situation in terms of pictures and/or diagrams*; 2) *know to relate the knowledge that one has to the problem situation*; 3) *know to think about what one is doing to set up an organized plan of steps*; 4) *know that abstracting/analyzing information from the problem situation aids in thinking about how best to approach the problem*; and 5) *know to brainstorm, splatter, and explore ideas about how to best approach the problem*. With the exception of the fifth metacognition, these characteristic metacognitions of the Cyclical conception were expressed by a small number of instructors in the Linear conception as well, but not enough to be considered characteristic for that conception. The remaining two metacognitions in Table 4-6 were not considered as characteristic of either conception.

There were some idiosyncrasies – those expressed by less than 10% of the 29 instructors – that deserve mentioning here. The metacognitions of *know that one should have a logical progression way of thinking or heuristic to help with setting up a solution* and *know that one should organize lots of sketches when setting up a solution* were only expressed by one or two instructors in the Cyclical conception. No instructor in the Linear conception expressed either of these metacognitions. On the other hand, one instructor in the Linear conception expressed the metacognition of *know that having a proper diagram serves as a check that one is not going astray in the solution*. Still another expressed the metacognitions of *know that setting up the problem should help* *one get a pretty good idea of where the solution is going*, *know that one should make some assumptions when setting up the solution*, and *know that one should understand how and why to solve problems*. It is interesting to note that although these metacognitions are all reasonable and fitting to problem solving, very few instructors actually mentioned them when describing the problem-solving process.

Monitoring involves metacognitions that are related to checking the progress of a solution to a problem. Table 4-7 shows the list of all of the *Monitoring* phrases that at least 10% of the 29 instructors expressed. The metacognitive phrases are grouped based on the similarities in their relations to particular components of the problem-solving process. The metacognitive phrases are also sequenced in the way that the instructors in the problem-solving process sequence the major components.

There was only one similarity in the way the two groups of instructors described the metacognitions. Large percentages of both the Linear and Cyclical instructors expressed the metacognition of *know to think explicitly about and justify reasoning that goes into the steps of a solution*. There was a difference, however, in the location within the problem-solving process that instructors in the two conceptions associated this metacognition. Instructors in the Linear conception associated this metacognition to the description of the major component **apply principles and concepts**. Instructors in the Cyclical conception, on the other hand, associated this metacognition to the description of the major component **experiment on an approach**. Although both major components are part of what could be considered as the execution stage of the problem-solving process, **apply principles and concepts** in the Linear conception was described as a component closer to the end of the execution stage, where as **experiment on an approach** in the Cyclical conception was described as a component closer to the beginning of the execution stage.

Some metacognitions that were characteristic for one conception were not expressed by a large enough percentage of the instructors in the other conception to also be considered as characteristic. Two characteristic metacognitions for the Linear conception were not characteristic of the Cyclical conception: 1) *know to justify the* *principle used*; and 2) *know to check the units of the equations used*. There were also four characteristic metacognitions for the Cyclical conception that were not characteristic of the Linear conception: 1) *realize when the solution is not progressing desirably*; 2) *know to evaluate the progress of the solution*; 3) *know to check the process of the solution*, and 4) *know to organize the solution*. Unsurprisingly, the first three metacognitions dealt explicitly with checking the performance of the solution process, and are indicative of the cyclical nature of this conception. In other words, if problem solving requires "going back" during the process, then consequently one would explicitly recognize the necessity of determining when one needs "go back". Since the Linear conception contains no backtracking, it is understandable that only a few of the instructors that expressed the Linear conception explicitly mentioned any of these three metacognitions. The remaining four metacognitions in Table 4-7 were not considered as characteristic of either conception.

There were some idiosyncrasies that deserve mentioning here. One instructor from both the Linear and Cyclical conception expressed the metacognition *know that having an approach helps one determine the most efficient mathematics*. Two instructors with the Linear conception also expressed the metacognition *know that one should check whether the equations are consistent with the principle to be used in the solution*. One instructor with the Cyclical conception expressed the metacognition *know that one could have many mistakes, analyses, struggles, and dead-ends in a solution*. It is interesting to note that although these metacognitions are all reasonable and fitting to problem solving, very few instructors actually mentioned them when describing the problem-solving process.

Evaluation involves the metacognitions that are related to checking the reasonableness of a solution to a problem. Table 4-8 shows the list of all of the *Evaluation* phrases that at least 10% of the 29 instructors expressed. The metacognitive phrases are listed in decreasing percentages.

There are similarities in the way the two groups of instructors described some of the metacognitions. Large percentages of both the Linear and Cyclical instructors expressed the metacognition of *know to think about whether the answer is reasonable with respect to the problem situation* and *know to think about whether the units of the answer is reasonable*. These two metacognitions constitute the bulk of what the instructors in both conceptions of problem solving expressed in terms of *Evaluation*. There was one metacognition characteristic of the Cyclical conception that was not also characteristic of the Linear conception: *know to check the relative magnitude of the answer*. The remaining two metacognitions in Table 4-8 were not considered as characteristic of either conception.

There were some idiosyncrasies that deserve mentioning here. One instructor with the Linear conception expressed the metacognition *know that one should find an alternative way to get an estimate of the reasonableness of the answer*. Another instructor with the Linear conception expressed the metacognition *know that one should make meaning of the answer*. It is interesting to note that although these metacognitions are all reasonable and fitting to problem solving, very few instructors actually mentioned them when describing the problem-solving process.

Table 4-6: Metacognitive phrasings about *planning* and the percentages of instructors who expressed each respective phrasing within each qualitatively different conception of the problem-solving process. The dark lines separate metacognitive phrasings that are related to similar components of the problem-solving process.

Planning	***Metacognitive Phrasing***	***Percentage (%) of instructors***		
		Linear (n = 22)	***Cyclical (n = 7)***	***Total (n=29)***
1	*Know to visualize the problem situation in terms of pictures and/or diagrams*	26	43	38
2	*Know to think explicitly about the problem situation in terms of the underlying physics*	55	71	59
3	*Know that having an understanding of the problem situation aids in the realization of what could be applied*	41	29	38
4	*Know to relate the knowledge that one has to the problem situation*	18	29	21
5	*Know to think about how to best approach the problem*	32	57	38
6	*Know to think about what one is doing to set up an organized plan of steps*	18	71	31
7	*Know that abstracting/analyzing information from the problem situation aids in thinking about how best to approach the problem*	18	29	21
8	*Know that realizing how to categorized the problem helps one set up an approach*	23	0	17
9	*Know to brainstorm, splatter, and explore ideas about how to best approach the problem*	0	57	14
10	*Know to realize what one knows and what one doe not know*	36	29	34
11	*Know that being clear about what is known and unknown makes problem solving easier and helps with making the necessary connections*	23	14	21
12	*Know to decide on a principle*	41	43	41

Table 4-7: Metacognitive phrasings about *monitoring* and the percentages of instructors who expressed each respective phrasing within each qualitatively different conception of the problem-solving process. The dark lines separate metacognitive phrasings that are related to similar components of the problem-solving process.

Monitoring	***Metacognitive Phrasing***	***Percentage (%) of instructors***		
		Linear (n = 22)	***Cyclical (n = 7)***	***Total (n=29)***
1	*Know to think explicitly about and justify reasoning that goes into the steps of a solution*	32	29	31
2	*Know to justify the principle used*	32	14	28
3	*Know to carefully analyze the steps*	14	0	10
4	*Know to think about which equation can be used*	9	14	10
5	*Know to make assumptions and see if the assumptions are reasonable*	27	14	24
6	*Know to decide on an assumption*	23	0	17
7	*Realize when the solution is not progressing desirably*	23	86	41
8	*Know to evaluate the progress of the solution*	14	29	17
9	*Know to check the process of the solution*	9	29	14
10	*Know to check the mathematics to make sure that the equations that one has can solve for the unknown*	14	0	10
11	*Know to check the units of the equations used*	45	14	38
12	*Know to organize the solution*	14	29	17

Table 4-8: Metacognitive phrasings about *monitoring* and the percentages of instructors who expressed each respective phrasing within each qualitatively different conception of the problem-solving process.

Evaluation	***Metacognitive Phrasing***	***Percentage (%) of instructors***		
		Linear (n = 22)	***Cyclical (n = 7)***	***Total (n=29)***
1	*Know to think about whether the answer is reasonable with respect to the problem situation*	41	57	45
2	*Know to think about whether the units of the answer is reasonable*	32	43	34
3	*Know to check the relative magnitude of the answer*	14	29	17
4	*Know to evaluate the solution*	14	14	14
5	*Know to pay attention to the significant figure of the answer*	14	0	10

Refined Explanatory Model: Answers to Sub-Question 2

The second sub-question for this convergent study is: When the sample of instructors is increased from 6 to 30,

Where appropriate, can the lack of detail in the problem-solving process be filled?

The Initial Explanatory Model did not include much detail of various components in the problem-solving process. Furthermore, it was often difficult to distill the relative importance of some of the items in the conception, and how representative these items are to the population of physics instructors. With the expansion of the sample, descriptions of the details expectedly increased, and facilitated the refinements necessary to converge on a more viable explanatory model.

Summary of the Details in the Refined Explanatory Model

The Refined Explanatory Model, with the explication of more details, provided a richer description of the components involved in the problem-solving process. Some of the seemingly idiosyncratic components in both conceptions in the initial model were either explicitly supported and thus included as an additional major component, or remained idiosyncratic and left out of the refined model. Both actions made the refined model more complete and less incoherent. The addition of the role of metacognition in the refined model provided a way to understand how physics instructors view the necessary thought processes that underlie problem solving. The inclusion of the role of metacognition in the Refined Explanatory Model made the implicit thought processes in the initial model explicit.

For example, under the Exploration and Trial and Error conception of the initial model, no explanations or extrapolations were given on how a problem solver is to accomplish the tasks of **using an understanding of physics to explore and come up with possible approaches**, **trying the possible approaches**, and **looking for errors**. In contrast, under the Cyclical conception of the refined model, the problem solver is to **brainstorm and explore to come up with possible approaches**, and at the same time *know to splatter and explore ideas about how to best approach the problem*, and *know to think about what one is doing to set up an organized plan of steps*. This requires **having and understanding of physics principles and concepts**, which in turn requires the problem solver to *know to related the knowledge that one has to the problem situation*. Then, the problem solver can **experiment on an approach**, by **figuring out what is needed** and **solve for what is being asked**. During the experimentation, the problem solver needs also to *know to evaluate the progress of the solution*, *know to check the process of the solution*, *realize when the solution is not progressing desirably*, and *know to think explicitly about and justify reasoning that goes into the steps of a solution*. This example, along with many others, shows how this convergent study has refined the Explanatory Model to be more complete.

Summary of the Role of Metacognition

The 30 physics instructors in this convergent study, as a whole, did not talk equally about the three different types of metacognition – *planning*, *monitoring*, and *evaluation*. In reality, the majority of metacognitive statements were about *planning*. This trend holds true even when the instructors were separated into groups based on the two conceptions of the problem-solving process. The instructors in the Cyclical conception, however, did on average have a higher percentage of the statements that were metacognitive than the instructors in the Linear conception.

Different phrasings of each type of metacognition were also identified. Overall, there were 12 phrases about *planning*, 12 phrases about *monitoring*, and 5 phrases about *evaluation*. Although there were similarities in the way the instructors in the two conceptions described these phrases, they did not focus on them in similar ways; some of the metacognitions that were characteristic for one conception were not characteristic of the other conception.

To sum up, this convergent study found that the Refined Explanatory Model of the Problem-Solving Process consists of two qualitatively different conceptions; a decision-making process that is Linear, and a decision-making process that is Cyclical. Each conception was refined from the Initial Explanatory Model to include not only more major components of the problem-solving process, but also more detailed descriptions of some of the major components. Furthermore, this convergent study also identified the role of metacognition within each conception of the problem-solving process. The richness of such details made the Refined Explanatory Model more coherent and better articulated than the Initial Explanatory Model.

Sub-Question 3: Viability of the Explanatory Model

This section will discuss the results pertaining to the third sub-question for this convergent study. The third sub-question for this convergent study is: When the sample of instructors is increased from 6 to 30,

Are the different conceptions of the problem-solving process really qualitatively different?

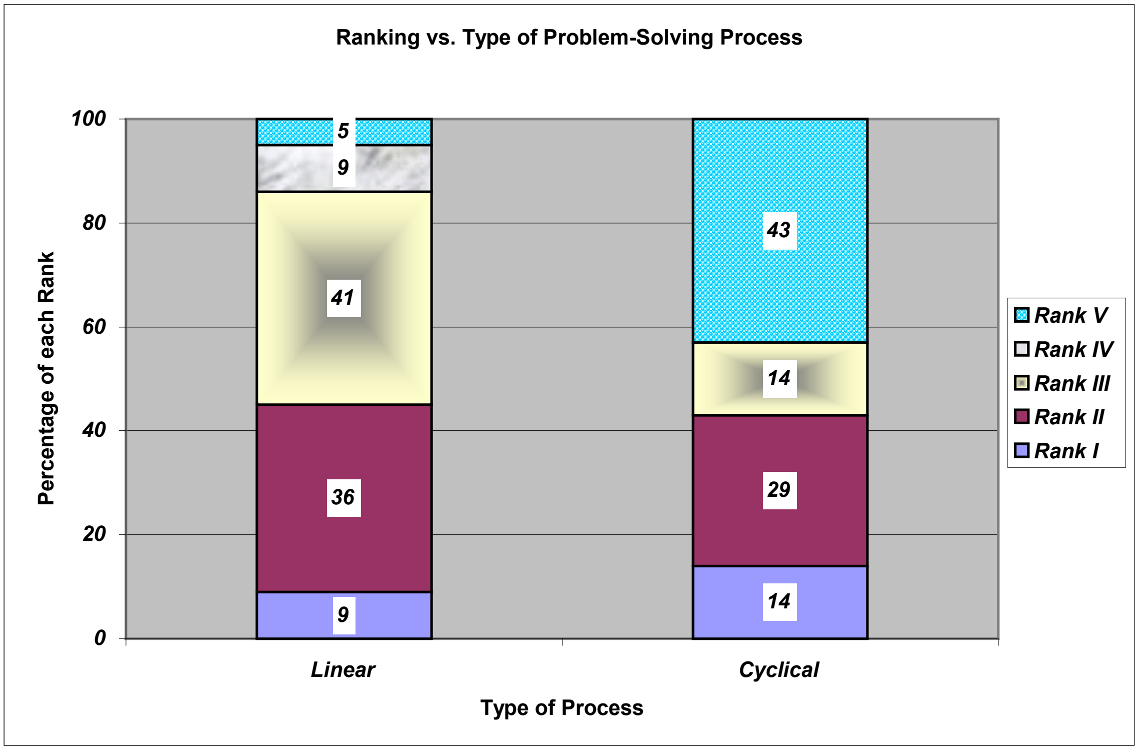
In order to provide validity to the previous discussions about the differences between the Linear and Cyclical conceptions of the problem-solving process in the Explanatory Model, consistency checks must be performed to verify the viability of the model. Since the qualitative differences were not undeniably large, it is necessary to determine the extent to which the qualitative differences were indeed different. This section will discuss the results of the consistency checks performed. As mentioned before, the purpose of these checks is to establish the legitimacy of the Linear and Cyclical conceptions of the problem-solving process as qualitatively different conceptions, rather than as mere artifacts of the data collection and analysis procedure.

Internal Consistency

The internal consistency of the results was checked by making comparisons with respect to the quantity and quality of the level of details in the individual concept maps. The expectation is that if the Linear and Cyclical conceptions of the problem-solving process are indeed qualitatively different, then the individual concept maps between the two conceptions will consequently consist of not only differing levels of detail, but also differing qualities in the detail.

The quantity and quality of the level of details was ranked based on a ranking scale described in Chapter 3. The criteria in the ranking scale were developed such that the individual concept maps can be sorted into groups, or ranks, where the maps in each group have more or less similar levels of details, both in quantity and in quality. The criteria for quantity of details are ***Requirements*** and ***Secondary Clarifications***. The criteria for quality of details are ***Reasons*** and ***Interconnections***.

Chart 4-2: Ranking of the concept maps in the Linear and Cyclical conceptions. The numbers represent the percentage of the concept maps within each conception that was ranked along the respective scale



In comparing the individual concept maps of the problem-solving process, it was reasonable to expect that those instructors who expressed the Cyclical conception will have more quality details than those instructors who expressed the Linear conception. The nature of the Cyclical process will consequently yield more interconnections between various parts within the problem-solving process. Furthermore, the necessity to "go back" also would conceivably lead those instructors to describe more details about how the problem solver could determine when to go back, and the rationale behind those decisions. Therefore, the distribution of the instructors who expressed the Cyclical conception of the problem-solving process should be skewed towards the higher end of the ranking scale. On the other hand, there is no overwhelming indication that the instructors who expressed the Linear conception of the problem-solving process should be skewed towards either end, thus the composition was expected to be somewhat normally distributed around the middle of the ranking scale. The results are shown in Chart 4-2.

The internal consistency check yielded results that are consistent with the expected distributions. Instructors that expressed the Linear conception of the problem-solving process overwhelmingly centered around Rank II and Rank III. Concept maps having 1 or 2 instances of each criterion – ***Requirement***, ***Reason***, ***Secondary Clarification***, and ***Interconnections*** – characterize these two ranks in the scale. Instructors that expressed the Cyclical conception of the problem-solving process, however, were indeed skewed towards the higher rank. It is characteristic of the concept maps in this rank to consist of more than 3 instances of each criterion. Therefore, it is reasonable to conclude that the Cyclical conception not only consists of more details than the Linear conception, but the details are of a higher quality. The characteristics of this internal consistency check are indications that the Linear and Cyclical conceptions of the problem-solving process are qualitatively different conceptions.

External Consistency

The external consistency of the analysis results was done by making comparisons with other sources of data from outside the set that was used to create the individual concept maps. This included data from various different parts of the background questionnaire, as well as data from parts of the interview transcripts that were not used in the creation of the individual problem-solving process concept maps. The expectation is that if the Linear and Cyclical conceptions of the problem-solving process are indeed qualitatively different, then the instructors between the two conceptions will also view other aspects of the problem solving differently. The external consistency checks were performed with respect to the following three sources of data:

From the Background Questionnaire,

1. Instructors' perceptions about the importance of quantitative problem solving
2. Instructors' perceptions about the importance of qualitative problem solving

From the interview situation dealing with Artifact Set III: Instructor Solutions

3. Instructors' perceptions about liking a particular example instructor solution

External Consistency Check 1

In the question involving the importance of Quantitative PS, there is a distinct difference in the distribution between the Linear and Cyclical conceptions (see Chart 4-3). For the instructors who expressed the Linear conception of the problem-solving process, 41% rated Quantitative PS as a **Very Important** goal for the calculus-based introductory physics course, 55% rated it as **Important**, and 4% rated it as **Somewhat Important**. For the instructors who expressed the Cyclical conception, however, 57% rated Quantitative PS as **Very Important**, and 43% rated it as **Important**. The relative percentages of **Very Important** and **Important** were reversed. Another difference is that although a small, but nevertheless apparent, percentage of the instructors in the

Linear conception rated Quantitative PS as **Somewhat Important**, no instructor in the Cyclical conception rated it as such.

External Consistency Check 2

In the question involving the importance of Qualitative PS, there is also a difference in the distribution between the Linear and Cyclical conceptions (see Chart 4-4). For the instructors who expressed the Linear conception, 36% rated Qualitative PS as a **Very Important** goal, 55% rated it as **Important**, and 9% rated it as **Somewhat Important**. For the instructors who expressed the Cyclical conception, however, 43% rated Qualitative PS as **Very Important**, and 57% rated it as **Important**. The relative percentages of the rating of **Important** were basically the same between the two conceptions. There is again a small, but apparent, percentage of the instructors in the Linear conception that rated Qualitative PS as **Somewhat Important**, where no instructor in the Cyclical conception rated it as such. This small percentage more or less makes up for the difference in the relative percentages of the **Very Important** rating between the Linear and Cyclical conceptions.

External Consistency Check 3

The results of the comparison exhibit a large difference between the instructors who expressed the Linear and those who expressed the Cyclical conceptions of the problem-solving process (see Chart 4-5). Half of the instructors who expressed the Linear conception also expressed their liking for IS II, which consists of a clear, step-by-step outline of the problem solution. On the other hand, almost three-fourth of the instructors who expressed the Cyclical conception expressed their liking for IS III, which consists of a qualitative analysis of the solution approach prior to the calculation. Since the nature of the Linear conception of the problem-solving process is in its step-by-step sequence of the solution, and the nature of the Cyclical conception is in its periodic re-analysis of the solution approach, these distributions are consistent with the notion that Linear and Cyclical conceptions of the problem-solving process are qualitatively different conceptions.

Chart 4-3: Rating of importance for the goal of Quantitative Problem Solving. The numbers represent the percentage of the instructors within each conception that rated along the respective scale.

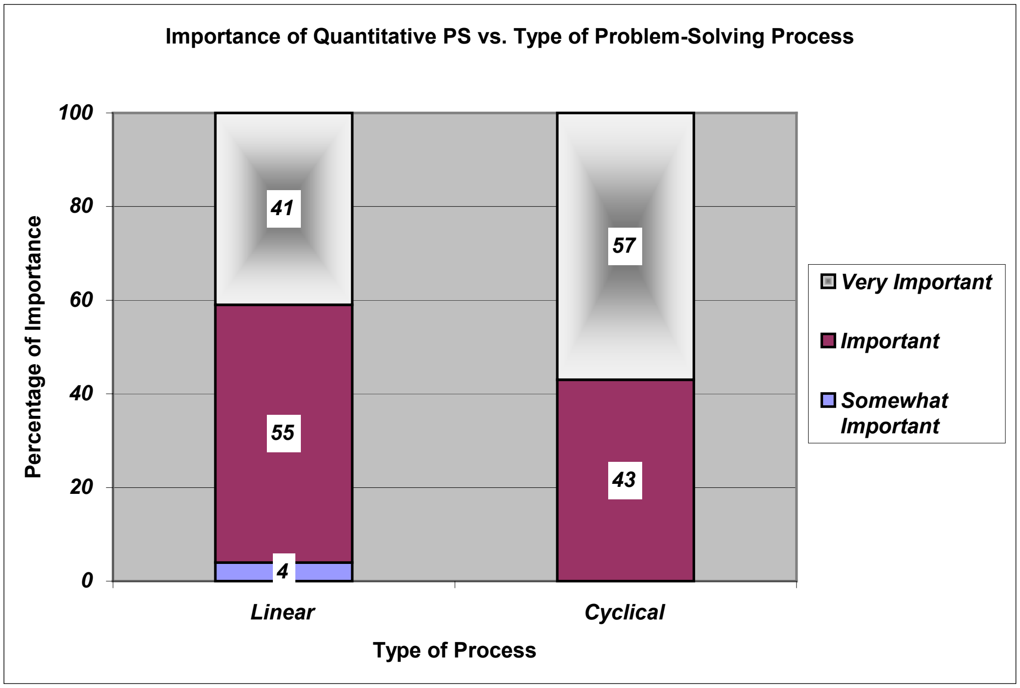


Chart 4-4: Rating of importance for the goal of Qualitative Problem Solving. The numbers represent the percentage of the instructors within each conception that rated along the respective scale.

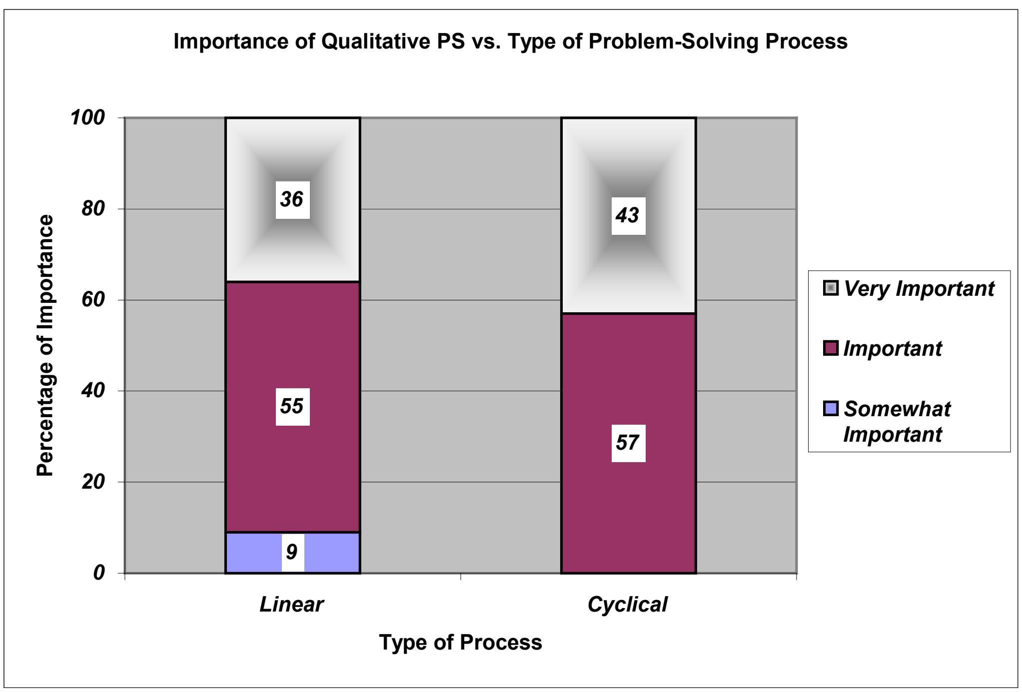
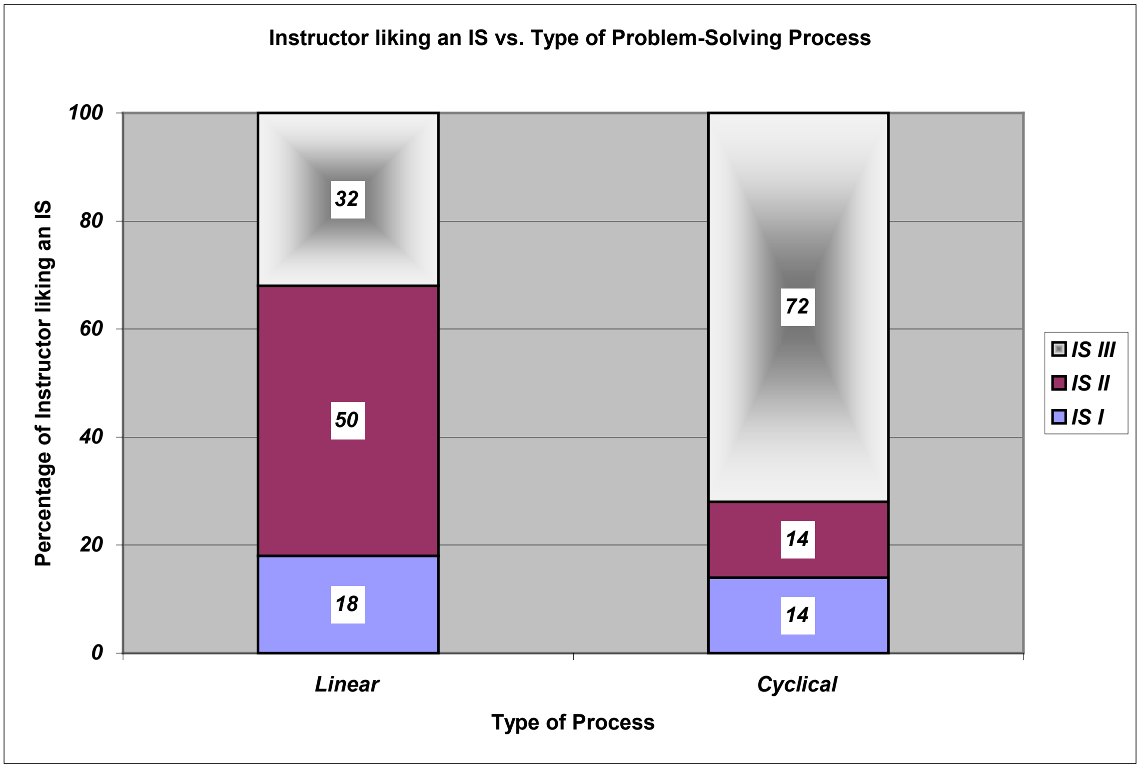


Chart 4-5: Liking for example Instructor Solutions. The numbers represent the percentage of the instructors within each conception that expressed their liking for a particular Instructor Solution.



Refined Explanatory Model: Answers to Sub-Question 3

The third sub-question for this convergent study is: When the sample of instructors is increased from 6 to 30,

Are the different conceptions of the problem-solving process really qualitatively different?

Although none of the consistency checks, both internally and externally, yielded overwhelmingly large differences between the instructors in the two conceptions, there existed a trend in the distributions. Taken individually, the result of each check alone would not be enough to make a judgment on whether the conceptions were qualitatively different. Taken as a whole, however, the results of each check yield a trend in the distribution that is hard to ignore.

The distribution in the internal consistency check, based on the ranking scale, shows that a much larger percentage of the individual concept maps in the Cyclical conception were of higher levels of quantity and quality in terms of their details. The distributions in external consistency check 1, on the rating of the importance of quantitative problem solving, shows that a larger percentage of the instructors in the Cyclical conception rated it as very important. The distributions in external consistency check 2, on the rating of the importance of qualitative problem solving, again shows that a larger percentage of the instructors in the Cyclical conception rated it as very important. The distributions in external consistency check 3, on liking a particular example instructor solution, shows once more that a larger percentage of the instructors in the Cyclical conception expressed their liking of the example instructor solution that most resembles an expert problem-solving framework.

Based on the results of these consistency checks, it is reasonable to say that the two different conceptions of the problem-solving process in the refined explanatory model are truly qualitatively different.

Summary

The purpose of this research program is to develop an explanatory model of physics instructors' conceptions about the teaching and learning of problem solving. Part of that initial explanatory model was the conception of Solving Physics Problems. This convergent study was conducted to refine that conception. This chapter described the refinements that were made based on an expansion of the sample of physics instructors. Three conceptions of the problem-solving process were identified: Linear, Cyclical, and Artistic. The first two conceptions had detailed descriptions of the processes involved, whereas the third was only described as being "different for different problems", and is the same as in the initial explanatory model. The initial explanatory model also consisted of the other two conceptions. They were described as "linear decision-making process" and "process of exploration and trial and error", respectively. As expected, an expansion in the sample resulted the expansion of the details. The refined explanatory model described in this chapter consisted of more coherent descriptions of the details involved in each conception of the problem-solving process. The qualitative differences between the Linear and Cyclical conceptions of the problem-solving process were further strengthened.

The most basic qualitative difference was in the nature of the conceptions. The Linear conception consisted of a step-by-step decision-making sequence, whereby the decisions made at each step was the correct one, as illustrated by the phrasing of the major component **recognize, decide on, and list the principles and concepts needed**. This sequence of correct decisions leads the problem solver to the correct solution, and no "backtracking" or "going back" is recognized as a necessity. The Cyclical conception consisted of an iterative decision-making process, whereby the decisions made at the beginning of the solution is treated as tentative, as illustrated by the phrasing of the major component **experiment on an approach**. This experimentation requires the problem solver to explicitly check the progress of the solution, and "backtracking" or "going back" is recognized as a pertinent and necessary part of the problem-solving process. On another level, the role of metacognition was also found to be different.

The qualitative differences between the instructors who expressed the Linear conception and those who expressed the Cyclical conception of the problem-solving process were checked for consistency, both internally and externally. These consistency checks yielded results that also exhibited differences between the two conceptions. Although the differences were small, all of the results pointed in the direction in support of the finding that Linear and Cyclical conceptions are qualitatively different conceptions of the problem-solving process. In other words, the small but consistent trend in both the internal and external consistency checks provide further evidence that the differences between the Linear and Cyclical conceptions of the problem-solving process are not mere artifacts of the data collection and analysis procedure, but are indeed qualitative differences.