Developing a Useful Instrument to Assess Student Problem Solving

Jennifer L. Docktor
Ken Heller

University of Minnesota
Physics Education Research & Development Group
http://groups.physics.umn.edu/physed
Problem solving is one of the primary teaching goals, teaching tools, and evaluation techniques of physics courses.

The goal is to develop a robust instrument to assess students’ written solutions to physics problems, and obtain evidence for reliability and validity.

The instrument should be general
- not specific to instructor practices or techniques
- applicable to a range of problem topics and types

This talk describes a test of the utility of the rubric
- The rubric gives useful information to focus instruction
- The rubric gives information to improve problem construction

Problem solving is an important part of learning physics.

Despite this, there is no standard way to evaluate problem solving that is VALID, RELIABLE, and EASY TO USE.

The goal of this project is...
- The instrument should be general; independent of instructor practices and applicable to a range of different topics in physics and types of problems

This talk describes a test of the utility of the rubric, from its application to students’ test solutions from a semester-long course.

I’ll give you the “punch line” from the start...we find the rubric gives useful information that can be used to direct instruction, and gives some information about the construction of problems.
This is the instrument we’re working on, at a glance. It takes the form of a rubric (which is a table or grid). It identifies five problem-solving sub-skill categories and defines performance levels for each category by a score and the criteria met to attain that score (empty boxes in this picture). If you want to see the rubric in its entirety, visit my website.

The categories are based on the research literature from cognitive psychology and physics education, and are intended to be somewhat independent. The idea is that a student receives a separate score for each category, and this will give a more detailed description of a student’s strengths and weaknesses in order to direct instruction.

For example, a student could identify appropriate physics principles and concepts to apply to the problem (physics approach) but have difficulty applying it to the specific conditions in the problem (specific application of physics). Or a student could have the physics correct, but get hung up on the math procedures.

We want this to be as simple as possible. We’re looking for the MINIMUM number of categories that will still include relevant aspects of problem solving, and the MINIMUM number of scores that will still give enough information to improve instruction. And we don’t want it to require a lot of training.
Rubric Scores (in general)

<table>
<thead>
<tr>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete &amp; appropriate</td>
<td>Minor omission or errors</td>
<td>Parts missing and/or contain errors</td>
<td>Most missing and/or contain errors</td>
<td>All inappropriate</td>
<td>No evidence of category</td>
</tr>
</tbody>
</table>

**NOT APPLICABLE (NA):**

<table>
<thead>
<tr>
<th>NA - Problem</th>
<th>NA - Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not necessary for this problem (i.e. visualization or physics principles given)</td>
<td>Not necessary for this solver (i.e. able to solve without explicit statement)</td>
</tr>
</tbody>
</table>

To make the rubric easier to apply, all categories are phrased in the same way / using consistent language. (i.e. a “5” means complete and appropriate across all categories)

In General, the rubric scores range from complete and appropriate (5) to minor and more serious errors (4-1) and all incorrect/missing.

The next two categories represent NA or “not applicable”. The NA Problem category means that a particular skill was not necessary for the problem, The NA Solver category means that based on the overall solution, it was not necessary for the solver to explicitly write down that step in the problem.
Calculus-Based Course for Science & Engineering @ UMN

- 4 Tests during the semester
- Problems graded in the usual way by teaching assistants
- After they were graded, I used the rubric to evaluate 8 problems spaced throughout the semester
  - Approximately 150 student solutions per problem

(say?) This course has a fall term enrollment of 930 students, split into four sections of ~230. I collected copies of tests from two of the sections that had the same instructor. Each test had two open-ended problems on it that were graded in a usual way by TAs (assigning partial credit and a single overall numerical score).

(don’t say?) Since teaching assistants were responsible for making test copies for their classes, I received an average of 150 papers from each section per problem (instead of 230). 300 x 8 test problems is about 2400 papers scored using the rubric.
Example Test Questions

A block of mass $m = 3\, \text{kg}$ and a block of unknown mass $M$ are connected by a massless rope over a frictionless pulley, as shown below. The kinetic frictional coefficient between the block $m$ and the inclined plane is $\mu_k = 0.17$. The plane makes an angle $30^\circ$ with horizontal. The acceleration, $a$, of the block $M$ is $1\, \text{m/s}^2$ downward.

(A) If the block $M$ drops by a distance $h$, how much work, $W$, is done on the block $m$ by the tension in the rope? Answer in terms of known quantities. [15 points]

---

NUMERIC

A block of mass $m = 3\, \text{kg}$ and a block of unknown mass $M$ are connected by a massless rope over a frictionless pulley, as shown below. The kinetic frictional coefficient between the block $m$ and the inclined plane is $\mu_k = 0.17$. The plane makes an angle $30^\circ$ with horizontal. The acceleration, $a$, of the block $M$ is $1\, \text{m/s}^2$ downward.

(A) Draw free-body diagrams for both masses. [5 points]
(B) Find the tension in the rope. [5 points]
(C) If the block $M$ drops by $0.5\, \text{m}$, how much work, $W$, is done on the block $m$ by the tension in the rope? [15 points]

---

SYMBOLIC

Same professor teaching two sections of the same course; sometimes gave the same exam questions to both sections, sometimes they were slightly modified. In this test (#3) they were slightly different. Highlighted in red – numeric versus symbolic, and FBD prompt.
Grader Scores

**Numeric, prompted:**
Several people received the full number of points, some about half.

**Symbolic:**
Fewer students could follow through to get the correct answer.

What can you tell from this?
Top graph: Some people got it, some got about half; the blip at 5 points is those who only drew a FBD and nothing else (worth 5 points)
Fewer people could follow through to get the right answer (2nd graph)
Rubric Scores

- **Useful Description:** Free-body diagram
- **Physics Approach:** Deciding to use Newton’s 2nd Law
- **Specific Application:** Correctly using Newton’s 2nd Law
- **Math Procedures:** solving for target
- **Logical Progression:** clear, focused, consistent

Each category represented by a different color; graphs represent frequency of scores in each category of the rubric.

For these questions, useful description meant drawing FBD and assigning symbols for quantities, approach meant...

Overall – graphs look very similar; Categories shifted up in numeric question

Point out differences: FBD prompted in numeric version (top graph); Logical Progression lower in symbolic question, specific application lower for numeric question

Common Specific Application errors: missing or extra force terms; sum to zero instead of ma; vector components, sign errors,...
Findings about the Problem Statement

- Both questions exhibited similar problem solving characteristics shown by the rubric.
  
  However
  
  - prompting appears to mask a student’s inclination to draw a free-body diagram
  - the symbolic problem statement might interfere with the student’s ability to construct a logical path to a solution
  - the numerical problem statement might interfere with the student’s ability to correctly apply Newton’s second law

- In addition, the numerical problem statement causes students to manipulate numbers rather than symbols

In numeric question, students often solved numerically for each force term and then summed the numbers to get a net force.
Findings about the Rubric

- The rubric provides significantly more information than grading that can be used for coaching students
  - Focus instruction on physics, math, clear and logical reasoning processes, etc.

- The rubric provides instructors information about how the problem statement affects students’ problem solving performance
  - Could be used to modify problems

This part of the study focused on the utility or usefulness of the rubric

Rubric indicates areas of student difficulties for an entire class; target instruction to physics, math, or logical reasoning

Have to be careful when interpreting rubric scores and also look at the way the problem is stated; rubric scores can reflect aspects of the problem statement (such as when the description is unnecessary) and it’s possible students’ natural problem solving behavior is masked.
<table>
<thead>
<tr>
<th>References</th>
<th><a href="http://groups.physics.umn.edu/phyled">http://groups.physics.umn.edu/phyled</a> <a href="mailto:docktor@physics.umn.edu">docktor@physics.umn.edu</a></th>
</tr>
</thead>
</table>

You can download the rubric and documentation at the UMN PER website (by clicking on my name) or send me an e-mail.
Rubric Category Descriptions

- **Useful Description**
  - organize information from the problem statement symbolically, visually, and/or in writing.

- **Physics Approach**
  - select appropriate physics concepts and principles to use

- **Specific Application of Physics**
  - apply physics approach to the specific conditions in problem

- **Mathematical Procedures**
  - follow appropriate & correct math rules/procedures

- **Logical Progression**
  - (overall) solution progresses logically; it is coherent, focused toward a goal, and consistent

(ADDITIONAL SLIDE – was cut out)

These sub-skills are based on those identified by research in cognitive psychology, especially the investigations of the differences between expert and novice problem solving processes

Reflect stages in the physics problem-solving process

Intended to be somewhat independent.
**Problem Characteristics that could Bias Problem Solving**

**Description:**
- Picture given
- Familiarity of context
- Prompts symbols for quantities
- Prompt procedures (i.e. Draw a FBD)

**Physics:**
- Prompts physics
- Cue focuses on a specific objects

**Math:**
- Symbolic vs. numeric question
- Mathematics too simple (i.e. one-step problem)
- Excessively lengthy or detailed math
<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>NA(Problem)</th>
<th>NA(Solver)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USEFUL DESCRIPTION</td>
<td>The description is</td>
<td>The description is useful but contains minor errors or</td>
<td>Parts of the description are useful but contain minor errors or</td>
<td>Parts of the description are useful but contain errors</td>
<td>Most of the description is not useful, missing, and/or contains errors</td>
<td>The entire description is not useful, missing, and/or contains errors</td>
<td>The solution does not include a description and it is necessary for this problem/ solver</td>
<td>A description is not necessary for this problem/ solver</td>
</tr>
<tr>
<td>PHYSICS APPROACH</td>
<td>The physics approach is appropriate and complete.</td>
<td>The physics approach contains minor errors or errors.</td>
<td>Some concepts and principles of the physics approach are missing and/or inappropriate.</td>
<td>Most of the physics approach is missing and/or inappropriate.</td>
<td>All of the chosen concepts and principles are inappropriate.</td>
<td>The solution does not indicate an approach, and it is necessary for this problem/ solver</td>
<td>An explicit physics approach is not necessary for this problem/ solver</td>
<td></td>
</tr>
<tr>
<td>SPECIFIC APPLICATION OF PHYSICS</td>
<td>The specific application of physics is appropriate and complete.</td>
<td>The specific application of physics contains minor errors or errors.</td>
<td>Parts of the specific application of physics are missing and/or contain errors.</td>
<td>Parts of the specific application of physics are missing and/or contain errors.</td>
<td>Most of the specific application of physics is missing and/or contain errors.</td>
<td>The entire specific application is inappropriate and/or contains errors.</td>
<td>The solution does not indicate an application of physics and it is necessary.</td>
<td>Specific application of physics is not necessary for this problem/ solver</td>
</tr>
<tr>
<td>MATHEMATICAL PROCEDURES</td>
<td>The mathematical procedures are appropriate and complete.</td>
<td>Appropriate mathematical procedures are used with minor errors or errors.</td>
<td>Parts of the mathematical procedures are missing and/or contain errors.</td>
<td>Parts of the mathematical procedures are missing and/or contain errors.</td>
<td>Most of the mathematical procedures are missing and/or contain errors.</td>
<td>All mathematical procedures are inappropriate and/or contain errors.</td>
<td>There is no evidence of mathematical procedures and they are necessary.</td>
<td>Mathematical procedures are not necessary for this problem/ solver</td>
</tr>
<tr>
<td>LOGICAL PROGRESSION</td>
<td>The entire solution is clear, focused, and logically connected.</td>
<td>The solution is clear and focused, and contains minor errors or errors.</td>
<td>Parts of the solution are unclear, uncertain, and inconsistent.</td>
<td>Parts of the solution are unclear, uncertain, and inconsistent.</td>
<td>Most of the solution is unclear, uncertain, and inconsistent.</td>
<td>The entire solution is unclear, uncertain, and inconsistent.</td>
<td>There is no evidence of logical progression, and it is necessary.</td>
<td>Logical progression is not necessary for this problem/ solver</td>
</tr>
</tbody>
</table>

2/15/2009 Jennifer Docktor, University of Minnesota

Version 4.4 of the rubric