INTRODUCTION

There has recently been growing interest in the dynamics of granular media such as sand or powders. An interesting property of mixtures of granular particles is their tendency to separate under conditions such as vertical shaking (Olsen and Rippie, 1964) and horizontal rotation (Oyama, 1939; Donald and Roseman, 1962; Bridgwater, 1976; Savage, 1993; Hill and Kakalios, 1994). Another variety of self-segregating behavior has been observed by Makse and co-workers (1996), where a binary mixture of sand and sugar will form alternating layers when poured near the closed edge of a “quasi-two-dimensional” Hele-Shaw cell. A theoretical model for this effect has been proposed by deGennes and Bouchaud (1996), extending the work of Bouchaud, Cates, Prakash, and Edwards (1994, 1995). Makse and co-workers (1997) have also reproduced the stratification effect with a numerical, computer simulation based on local height differences between rectangular particles. Current models describing avalanche segregation are explicitly for two-dimensional sand piles. The experimental studies are extended to three dimensions by systematically varying the plate width of the Hele-Shaw cell. The segregation process is also studied as a function of the flow rate of arriving particles. Furthermore, a computer simulation based on physical properties of particles has been implemented to reproduce the stratification effect.
EXPERIMENTAL TECHNIQUES

The Hele-Shaw cell (Hele-Shaw, 1898) consists of two vertical Plexiglas sheets 20 cm high and 30 cm long, mounted parallel to each other onto a Plexiglas base. One of the vertical sheets is bolted to the base. The distance between the vertical sheets is controlled by a 20 cm long strip-like spacer that closes one edge of the cell and two small block spacers at the opposite edge. By bolting the vertical plates together with various sized spacers, the plate separation is varied from 3 mm to 24 mm. A 50/50 mixture by mass of sand and sugar is poured near the closed edge of this cell, using a titration bulb with a rotating stopcock to control flow rate. The sand has been dyed blue/black for contrast in the photos without affecting the segregation process. Digital photos of experimental runs are taken with a monochrome CCD camera (Cohu 4910) in conjunction with a Scion LG-3 frame grabber and a Power Macintosh 7100/80. Data image analysis is performed with the public domain program Image from the NIH.

Figure 1: Digital image of the resulting sand pile formed by pouring a 50/50 sand/sugar mixture at 0.8 gm/sec near the closed, right edge of the Hele-Shaw cell (not shown) with a plate separation of 4 mm.

EXPERIMENTAL RESULTS

Figure 1 shows a digital image of the resulting sand pile formed by pouring a 50/50 sand/sugar mixture at 0.78 ± 0.08 gm/sec near the closed edge of the Hele-Shaw cell with a plate separation of 4 ± 0.5 mm. The first feature of note is the “dead-zone” of mixed sand and sugar at the lower right hand corner in figure 1. The size of this zone depends on both the plate separation and flow rate. In addition, the stratification is most apparent in the middle of the sand pile, with single species of sand and sugar collecting at the top and bottom of the pile.
respectively. Makse and co-workers (1996) also noted this “downhill segregation” with the larger, less dense sugar particles flowing further down the pile. Identical patterns have been observed when the mixture is poured in the center of the cell, indicating that segregation is not an edgewall effect.

Figure 2: Digital image (fig. 2a) of the segregated banding pattern near the top of the sand pile (indicated by the white box in fig. 1). Also shown is the average pixel intensity against position, obtained using the graphics program Image (fig. 2b). The high pixel values correspond to the dark regions of the image (high sand concentration) and the low pixel values indicate regions of high sugar (white) concentration. This data is then Fourier transformed to yield the structure function of the banding pattern (fig. 2c).

Previous theories on avalanche segregation have not discussed the influence of plate separation on the segregation process. Therefore, a systematic study of varying vertical plate separation in the Hele-Shaw cell has been completed. Two trends are observed as the plate separation is increased at a constant flow rate: the degree of segregation decreases as does the spacing between segregated bands. These trends are quantified through measurement of the structure function. Figure 2a shows a digital image of the banding pattern near the top of the sand pile, as indicated by the white box in figure 1. The program Image can plot the pixel intensity against the horizontal position. Using white sugar and black sand lets us correlate the pixel intensity to the concentration of sand near the transparent vertical plate of the Hele-Shaw cell. The resulting pixel intensity graph (fig. 2b) is fast Fourier transformed to yield the structure function for the banding pattern (fig. 2c).
The location of the peak in the plot of FFT amplitude against wave vector in figure 2c gives the average wavelength of the segregated bands, while the amplitude of the peak provides a quantitative measure of the degree of segregation. Both the degree of segregation and the average wavelength of the bands decrease in a nonlinear manner as the plate separation is increased. The FFT amplitude as a function of plate width is accurately described by either an exponential or power law relationship (fig. 3a and b).

Figure 3a: Log-linear plot of the peak amplitude in the structure function for the avalanche segregation banding pattern against plate separation of the Hele-Shaw cell for a mixture of sand and sugar. The solid line is a fit to the data with an exponential dependence on plate separation.

Figure 3b: A Log-log plot of the data in fig. 3a, indicating the equally good power law fit, where the power law exponent is ~ 1.6.

Ideally one would like to extend the horizontal scale for figures 3a and 3b in order to determine which functional form accurately describes the data. However, at smaller plate separations one can not force enough of the granular mixture into the Hele-Shaw cell at a uniform flow rate, while at larger plate separations the segregation effect disappears. Further experiments using alternate granular mixtures, for which the segregation effect persists at larger plate separations, are presently underway and should enable a determination of the functional form of the plate width dependence.

The width of the bands (wavelength) depends on the time interval between avalanches, which, in turn, depends on properties of the granular material (that determine the maximum angle of repose), the plate separation, and the flow rate. For a given plate separation, there is a maximum flow rate at which segregation occurs. For rates above this sharply defined maximum flow rate, a completely mixed sand pile results, due to a continuous flow regime.
A peculiar feature of the experiment that is unaccounted for in current theories is the phenomena of pairing between segregated bands of pure species at plate separations of 5 - 8 mm and flow rates of 0.75 - 1.00 g/s.

Figure 4: Example of pure species band pairing at a plate width of 6.5 mm and a flow rate of 0.832 g/s.

The observed behavior of the avalanching system described, indicates conditions at which segregation will occur. When plotting the factors governing avalanche segregation (flow rate and plate width) against each other, we find upper and lower nonlinear boundaries at which segregation ceases. This pseudo phase diagram involving plate width and flow rate quantifies the constraints needed in a system to produce segregation in disordered binary mixtures. Work is currently underway toward further understanding of the conditions governing avalanche segregation and completion of the previously mentioned phase diagram.

COMPUTER SIMULATION

Previous numerical simulations of granular media stratification have used a lattice grid of rectangular particles, where each particle must reside in a specific column. Movement of individual particles is determined by the height difference between adjacent columns, with the criteria dependent on the sizes of the moving particle and the stationary particle on which it rests. A physical simulation that incorporates material properties of the granular media, such as size, shape, momentum, and center coordinates has been developed in correlation with the experiment.

In the simulation, circular particles arrive near the closed edge of a two-dimensional, “virtual” Hele-Shaw cell with an initial momentum. Each particle bounces according to kinematic equations, losing a fixed percentage of momentum at each collision. When its momentum is below a critical value, its rolls until its momentum approaches zero. During the rolling phase, the particle loses a fixed percentage of its momentum each time it passes a stable position. (A stable position occurs when a particle’s horizontal, center of mass coordinate lies between two, simultaneous contact points.) Preliminary results yield a stratification pattern dependent upon a larger angle of repose for the pure species of large particles (fig. 5a). This corresponds to the local height difference criteria that Makse and coworkers used in previous simulations (1997). However, the slope variation is caused by differing initial momentum values for large and small particles. Stratification results when larger particles move down the hill and build a layer up from the bottom, while smaller particles stack up at the top of the pile.
and eventually avalanche over the large particle band. While these initial results are encouraging, the simulations do not reproduce the sharp nearly periodic segregated layers as in figure 1. Work is continuing in order to refine the simulation algorithm to include the effects of local variations in the critical angle of repose of the pile and improve the match between simulations and experiment.

Figure 5a: Overlapping piles of pure species indicate a higher angle of repose for the larger particles, which is a necessary condition for stratification.

Figure 5b: Resulting stratification pattern from a simulation of 10,000 particles with a 50/50 volume ratio of the large, gray particles to the small black particles.
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REFERENCES

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